The genetic system and algebras of projection operators

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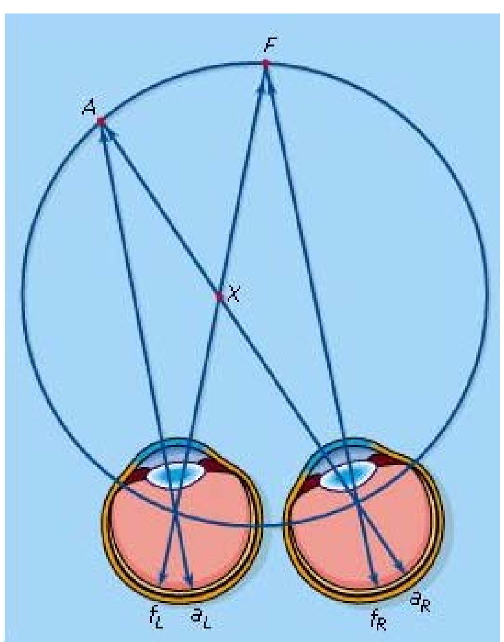
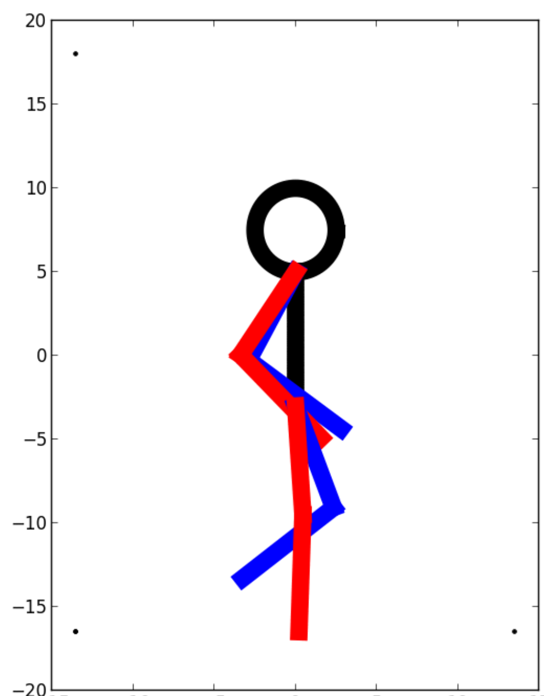
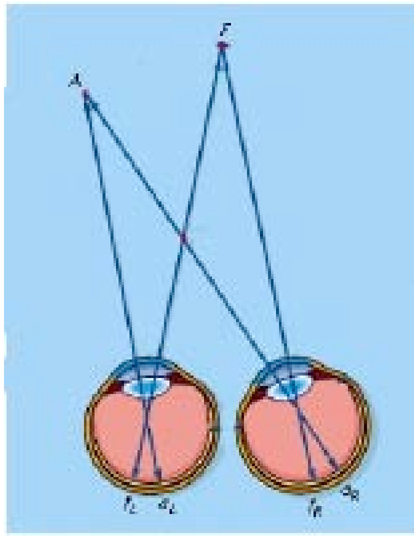


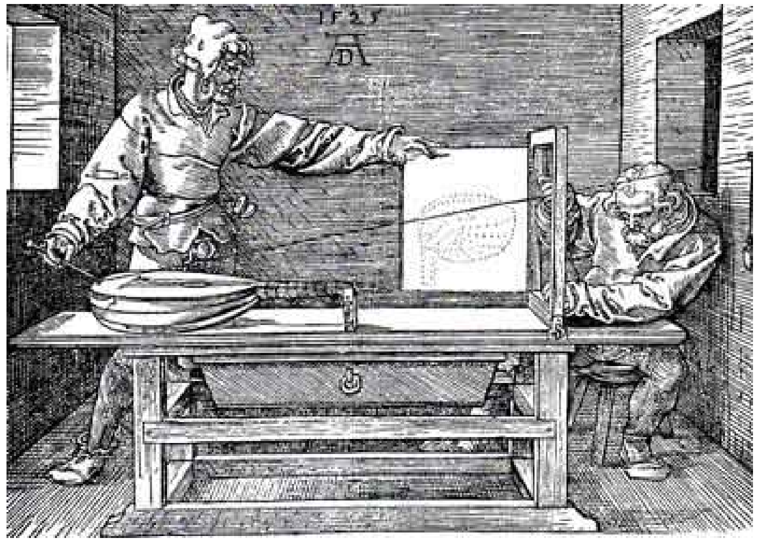


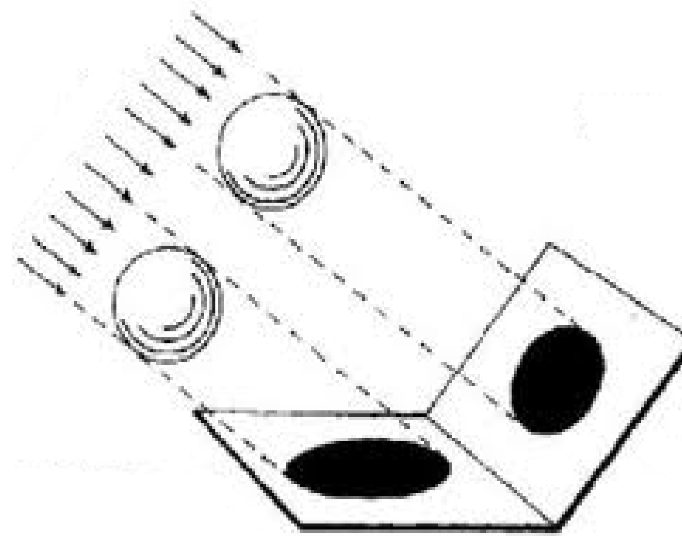
Science has led to a new understanding of life itself: **«Life is a partnership between genes and mathematics»** (Stewart I. Life's other secret: The new mathematics of the living world. 1999, New-York: Penguin).

But what kind of mathematics is a partner with the genetic code and defines the structure of living matter? This lecture shows that phenomenology of ensembles of molecular-genetic elements is connected with ensembles of **projection operators**, which are well-known in physics, informatics, chemistry, etc.

Information from the micro-world of genetic molecules dictates constructions in the macro-world of living organisms under strong noise and interference. This dictation is realized by means of unknown algorithms of multi-channel noise-immunity coding. For example, in accordance with Mendel's laws of independent inheritance of traits, colors of human skin, eye and hairs are genetically defined independently. So, each **living organism is an algorithmic machine of multi-channel noise-immunity coding**. To understand this machine we should use the theory of noise-immunity coding, which is based on **matrix representations** of digital information. Correspondingly we search mathematics of genetic systems in matrix representations of ensembles of genetic elements.



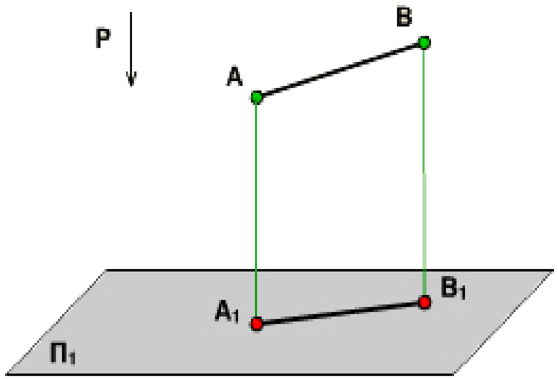




Our visual perception and a “projection method” in the drawing are based on projections of external objects on retina and a drawing plane. In mathematics, such operations of projections are expressed by means of **square matrices**, which are called “projection operators” (or “projectors”).

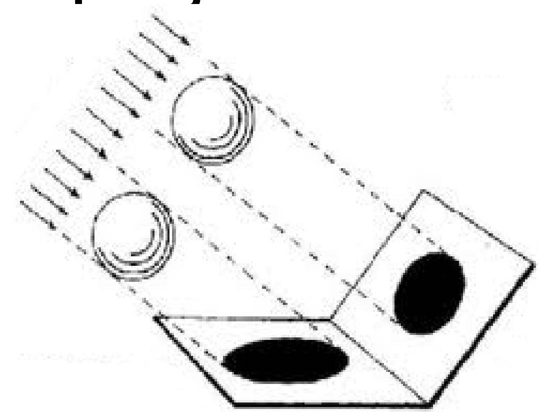
The following matrix P is an example of a projector, which makes a projection of vector [x, y, z] on the plane [x, y, 0]:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad P \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}.$$



A necessary and sufficient condition that a matrix P is a projection operator is given by the criterion: $\mathbf{P^2 = P}$. Many matrices in this lecture will satisfy this criterion.

Projection operators are used widely in math, physics (including quantum mechanics), chemistry, informatics, logics, etc. Two kinds of projectors exist: orthogonal projectors are expressed by symmetric matrices; **oblique projectors** – by asymmetric matrices (they are less studied).



This lecture shows that **projectors can be also used in studying and modeling properties of the genetic code system and many inherited biological ensembles** including ensembles of cyclic processes, phyllotaxis patterns, etc.

EXPLANATION ABOUT GENETIC MATRICES R_8 AND H_8

Theory of noise-immunity coding is based on matrix methods. For example, matrix methods allow transferring high-quality photos of Mar’s surface via millions of kilometers of strong interference. In particularly, Kronecker families of Hadamard matrices are used for this aim.

$$H_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}; \quad H_2^{(2)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}; \quad H_2^{(3)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

Here (n) means a Kronecker power.

By analogy with theory of noise-immunity coding, the 4-letter alphabet of RNA (adenine A, cytosine C, guanine G and uracil U) can be represented in a form of a (2*2)-matrix [C U; A G] as a kernel of a Kronecker family of matrices [C U; A G]$^{(N)}$, where (N) means a Kronecker power. The third Kronecker power of this alphabetic (2*2)-matrix gives the (8*8)-matrix of 64 triplets disposed in a strong order. These 64 triplets encode amino acids of proteins.

$$\begin{vmatrix} C & U \\ A & G \end{vmatrix}^{(3)} =$$

CCC	CCU	CUC	CUU	UCC	UCU	UUC	UUU
CCA	CCG	CUA	CUG	UCA	UCG	UUA	UUG
CAC	CAU	CGC	CGU	UAC	UAU	UGC	UGU
CAA	CAG	CGA	CGG	UAA	UAG	UGA	UGG
ACC	ACU	AUC	AUU	GCC	GCU	GUC	GUU
ACA	ACG	AUA	AUG	GCA	GCG	GUA	GUG
AAC	AAU	AGC	AGU	GAC	GAU	GGC	GGU
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

Two first positions of each triplet is termed as a "root" of the triplet. The phenomenological fact is that the set of 64 triplets is divided by the nature into two equal subsets with 32 triplets in each. The first subset contains 32 triplets with "strong roots" CC, CU, CG, AC, UC, GC, GU, GG (it means that all triplets, which have one of these roots, encode the same amino acid). The second subset contains 32 triplets with "weak roots" CA, AA, AU, AG, UA, UU, UG, GA (it means that all triplets, which have one of these roots, encode not the same amino acid).

Whether any symmetry exists in a disposition of triplets with strong and weak roots in the matrix of triplets $[C\ U;\ A\ G]^{(3)}$ constructed formally ?

THE STANDARD CODE	
8 subfamilies of triplets with strong roots («black triplets») and amino acids, which are encoded by them	8 subfamilies of triplets with weak roots («white triplets») and amino acids, which are encoded by them
CCC, CCT, CCA, CCG → Pro	CAC, CAT, CAA, CAG → His, His, Gln, Gln
CTC, CTT, CTA, CTG → Leu	AAC, AAT, AAA, AAG → Asn, Asn, Lys, Lys
CGC, CGT, CGA, CGG → Arg	ATC, ATT, ATA, ATG → Ile, Ile, Ile, Met
ACC, ACT, ACA, ACG → Thr	AGC, AGT, AGA, AGG → Ser, Ser, Arg, Arg
TCC, TCT, TCA, TCG → Ser	TAC, TAT, TAA, TAG → Tyr, Tyr, Stop, Stop
GCC, GCT, GCA, GCG → Ala	TTC, TTT, TTA, TTG → Phe, Phe, Leu, Leu
GTC, GTT, GTA, GTG → Val	TGC, TGT, TGA, TGG → Cys, Cys, Stop, Trp
GGC, GGT, GGA, GGG → Gly	GAC, GAT, GAA, GAG → Asp, Asp, Glu, Glu
THE VERTEBRATE MITOCHONDRIAL CODE	
CCC, CCT, CCA, CCG → Pro	CAC, CAT, CAA, CAG → His, His, Gln, Gln
CTC, CTT, CTA, CTG → Leu	AAC, AAT, AAA, AAG → Asn, Asn, Lys, Lys
CGC, CGT, CGA, CGG → Arg	ATC, ATT, ATA, ATG → Ile, Ile, Met, Met
ACC, ACT, ACA, ACG → Thr	AGC, AGT, AGA, AGG → Ser, Ser, Stop, Stop
TCC, TCT, TCA, TCG → Ser	TAC, TAT, TAA, TAG → Tyr, Tyr, Stop, Stop
GCC, GCT, GCA, GCG → Ala	TTC, TTT, TTA, TTG → Phe, Phe, Leu, Leu
GTC, GTT, GTA, GTG → Val	TGC, TGT, TGA, TGG → Cys, Cys, Trp, Trp
GGC, GGT, GGA, GGG → Gly	GAC, GAT, GAA, GAG → Asp, Asp, Glu, Glu

Figure shows triplets with strong roots (black color) and weak roots (white color) in the Standard Genetic Code and the Vertebrate Mitochondrial Genetic Code

It should be noted that a huge quantity $64! \approx \mathbf{10^{89}}$ of variants exists for dispositions of 64 triplets in the (8*8)-matrix. For comparison, the modern physics estimates time of existence of the Universe in $\mathbf{10^{17}}$ seconds. It is obvious that an accidental disposition of the 20 amino acids and the corresponding triplets in a (8*8)-matrix will give almost never any symmetry.

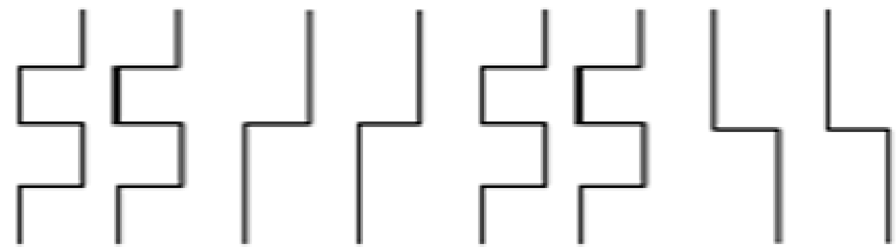
But unexpectedly the phenomenological disposition of the 32 triplets with strong roots (black color) and the 32 triplets with weak roots (white color) has a symmetric character: 1) both quadrants along each of diagonals are identical by their mosaic; 2) the upper half and the lower half of the matrix are mirror-anti-symmetric to each other in its colors: any pair of cells, disposed by mirror-symmetrical manner in these halves, possesses the opposite colors.

$$\begin{vmatrix} C & U \\ A & G \end{vmatrix}^{(3)} =$$

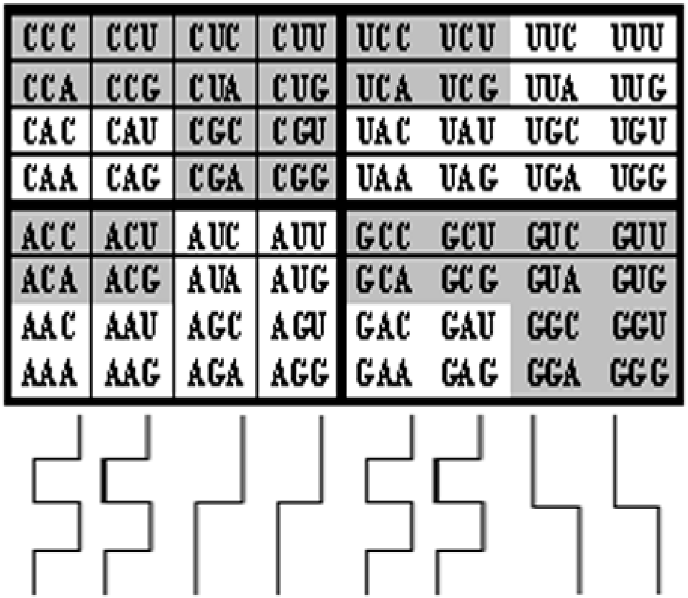
CCC	CCU	CUC	CUU	UCC	UCU	UUC	UUU
CCA	CCG	CUA	CUG	UCA	UCG	UUA	UUG
CAC	CAU	CGC	CGU	UAC	UAU	UGC	UGU
CAA	CAG	CGA	CGG	UAA	UAG	UGA	UGG
ACC	ACU	AUC	AUU	GCC	GCU	GUC	GUU
ACA	ACG	AUA	AUG	GCA	GCG	GUA	GUG
AAC	AAU	AGC	AGU	GAC	GAU	GGC	GGU
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

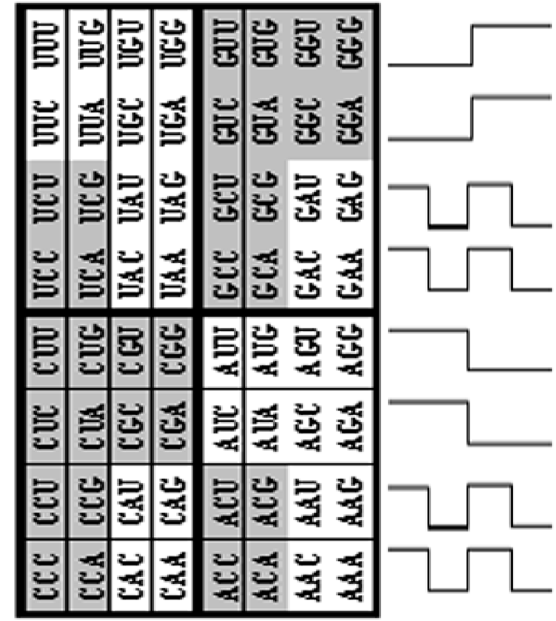
$[C\ U;\ A\ G]^{(3)} =$

CCC	CCU	CUC	CUU	UCC	UCU	UUC	UUU
CCA	CCG	CUA	CUG	UCA	UCG	UUA	UUG
CAC	CAU	CGC	CGU	UAC	UAU	UGC	UGU
CAA	CAG	CGA	CGG	UAA	UAG	UGA	UGG
ACC	ACU	AUC	AUU	GCC	GCU	GUC	GUU
ACA	ACG	AUA	AUG	GCA	GCG	GUA	GUG
AAC	AAU	AGC	AGU	GAC	GAU	GGC	GGU
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG



The most important fact is that a mosaic character of each of columns corresponds to an **<u>odd meander-like function</u>**. But such odd meander-like functions are well-known in theory of signal processing under the name “**<u>Rademacher functions</u>**”.

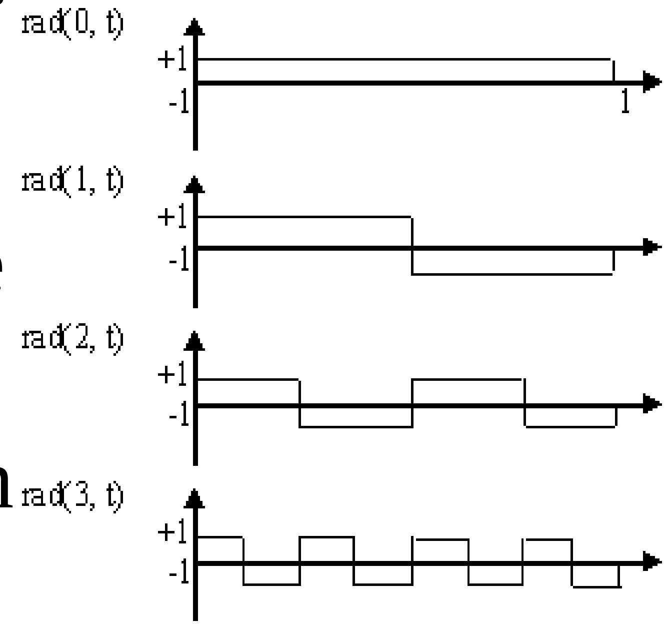




Examples of Rademacher functions:

$$r_n(t) = \mathrm{sign}(\sin 2^n \pi t)$$

Rademacher functions contain only elements “+1” and “-1”. Each of the matrix columns presents one of the Rademacher functions if each black (white) cell is interpreted such that it contains the number +1 (−1).



$$\begin{bmatrix} C & U \\ A & G \end{bmatrix}^{(3)} = \begin{bmatrix} CCC & CCU & CUC & CUU & UCC & UCU & UUC & UUU \\ CCA & CCG & CUA & CUG & UCA & UCG & UUA & UUG \\ CAC & CAU & CGC & CGU & UAC & UAU & UGC & UGU \\ CAA & CAG & CGA & CGG & UAA & UAG & UGA & UGG \\ ACC & ACU & AUC & AUU & GCC & GCU & GUC & GUU \\ ACA & ACG & AUA & AUG & GCA & GCG & GUA & GUG \\ AAC & AAU & AGC & AGU & GAC & GAU & GGC & GGU \\ AAA & AAG & AGA & AGG & GAA & GAG & GGA & GGG \end{bmatrix}$$

$$R_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

Here we show a transformation of the mosaic genomatrix [C U; A G]$^{(3)}$ into the numeric matrix R_8 in the result of such replacements of triplets with strong and weak roots by means of numbers "+1" and "-1" correspondingly. This numeric matrix R_8 is called the "Rademacher form" of the genetic matrix of triplets [C U; A G]$^{(3)}$ or briefly the **"Rademacher matrix" R_8**.

Taking into account another phenomenological fact about a unique status of uracil U (which is replaced by thymine T in DNA), a simple U-algorithm exists, which transforms the matrix [C U; A G]$^{(3)}$ into the matrix [C T; A G]$^{(3)}$ with a new black-and-white mosaic (a triplet changes its color, if it has U in its odd position; this algorithm can be described below [Petoukhov, 2008]). This new mosaic corresponds to mosaic of one of **Hadamard matrices H_8**.

CCC	CCU	CUC	CUU	UCC	UCU	UUC	UUU
CCA	CCG	CUA	CUG	UCA	UCG	UUA	UUG
CAC	CAU	CGC	CGU	UAC	UAU	UGC	UGU
CAA	CAG	CGA	CGG	UAA	UAG	UGA	UGG
ACC	ACU	AUC	AUU	GCC	GCU	GUC	GUU
ACA	ACG	AUA	AUG	GCA	GCG	GUA	GUG
AAC	AAU	AGC	AGU	GAC	GAU	GGC	GGU
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

Hadamard matrices also contain only entries "+1" and "-1". Columns of a Hadamard matrix form a complete orthogonal set of Walsh functions. If each black (white) cell of this symbolic matrix is interpreted as a container of number +1 (−1), then the **Hadamard matrix H_8** arises. Each of columns of the numeric matrix H_8 presents one of the Walsh functions.

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

H_8 =

1	-1	1	-1	-1	1	1	-1
1	1	1	1	-1	-1	1	1
-1	1	1	-1	1	-1	1	-1
-1	-1	1	1	1	1	1	1
1	-1	-1	1	1	-1	1	-1
1	1	-1	-1	1	1	1	1
-1	1	-1	1	-1	1	1	-1
-1	-1	-1	-1	-1	-1	1	1

Hadamard matrices are intensively explored in digital signal processing including noise-immunity coding.

For example, codes based on Hadamard matrices have been used on spacecrafts «Mariner» and «Voyadger», which allowed obtaining high-quality photos of Mars, Jupiter, Saturn, Uranus and Neptune in spite of the distortion and weakening of the incoming signals.

Hadamard matrices are used to create quantum computers, which are based on Hadamard gates. They are used in quantum mechanics in the form of unitary operators.

Now we reveal and study the connection of the genetic code with a special kind of Hadamard matrices.

The main mathematical objects of the lecture will be these two (8*8)-matrices, which reflect phenomenological properties of the molecular-genetic ensembles: the Rademacher matrix R_8 and the Hadamard matrix H_8.

$$R_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix} \qquad H_8 = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

What secrets of the genetic code and living matter are hidden in these mosaic matrices? Let’s study these matrices using their “Rademacher decomposition” and “Walsh decomposition” correspondingly.

$$R_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix} \qquad H_8 = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

Each of matrices R_8 and H_8 is a sum of 8 sparse matrices, in which only one of columns is non-zero (all these columns coincide with Rademacher or Walsh functions):

$R_8=s_0+s_1+s_2+s_3+s_4+s_5+s_6+s_7$; $H_8=u_0+u_1+u_2+u_3+u_4+u_5+u_6+u_7$

$$R_8 = \begin{vmatrix} 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{vmatrix} +$$

$$\begin{vmatrix} 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0 \end{vmatrix} +$$

$$\begin{vmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&-1&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{vmatrix}$$

$$H_8 = \begin{vmatrix} 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&-1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{vmatrix} +$$

$$\begin{vmatrix} 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&-1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0 \end{vmatrix} +$$

$$\begin{vmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0 \end{vmatrix} + \begin{vmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1 \end{vmatrix}$$

$$R_8 = \begin{pmatrix}1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\end{pmatrix} +$$

$$\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&-1&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0&0\end{pmatrix} +$$

$$\begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&-1&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$H_8 = \begin{pmatrix}1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&-1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\end{pmatrix} +$$

$$\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&-1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&-1&0&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0&0\end{pmatrix} +$$

$$\begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&-1&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&0\end{pmatrix} + \begin{pmatrix}0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&0&1\end{pmatrix}$$

In these decompositions of $\mathbf{R_8}=s_0+s_1+s_2+s_3+s_4+s_5+s_6+s_7$ and $\mathbf{H_8}=u_0+u_1+u_2+u_3+u_4+u_5+u_6+u_7$, **every of 16 sparse matrices** $s_0, \ldots, s_7, u_0, \ldots, u_7$ **is <u>a projection operator</u>** because it satisfies the criterion $\mathbf{P^2 = P}$. It means that genetic matrices $\mathbf{R_8}$ and $\mathbf{H_8}$ are **sums of oblique projectors; the genetic system is connected with projectors**.

Now let us show how these "genetic" projectors allow modeling genetically inherited bio-ensembles.

INHERITED ENSEMBLES OF BIOLOGICAL CYCLES

Any living organism is a huge ensemble of inherited cyclic processes, which form a hierarchy at different levels. Even every protein is involved in a cycle of its "birth-death," because after a certain time it breaks down into its constituent amino acids and they are then collected into a new protein. According to chrono-medicine and bio-rhythmology, various diseases of living bodies are associated with disturbances (dys-synchronization) in these cooperative ensembles of biocycles.

It is known that mathematical cyclic groups are useful to model natural cyclic processes. But **combinations of the considered genetic projectors lead to a great number of cyclic groups**.

For example, take sum of two projectors s_0 and s_2:

$$\mathbf{s_0+s_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exponentiation $(2^{-0.5}*(s_0+s_2))^N$ gives a **cyclic group with its period 8**: $(2^{-0.5}*(s_0+s_2))^{\mathbf{N}} = (2^{-0.5}*(s_0+s_2))^{\mathbf{N+8}}$ (here N=1, 2, 3, …).

Iterative actions of this operator $Y = (2^{-0.5}*(s_{\mathbf{0}}+s_{\mathbf{2}}))$ on an arbitrary 8-dimensional vector $X=[x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7]$ give a cyclic set of vectors, in which only two coordinates with appropriate indexes **0** and **2** are cyclic changed, all other coordinates are equal to 0:

$$X*Y^1 = 2^{-0.5}*[(x_0+x_1-x_2-x_3+x_4+x_5-x_6-x_7),\ 0,\ (x_0+x_1+x_2+x_3-x_4-x_5-x_6-x_7),\ 0, 0, 0, 0,$$

$$X*Y^2 = [(x_4-x_3-x_2+x_5),\ 0,\ (x_0+x_1-x_6-x_7),\ 0, 0, 0, 0, 0]$$

$$X*Y^3 = 2^{-0.5}*[(x_4-x_1-x_2-x_3-x_0+x_5+x_6+x_7),\ 0,\ (x_0+x_1-x_2-x_3+x_4+x_5-x_6-x_7),\ 0, 0, 0, 0,$$

$$X*Y^4 = [(x_6-x_1-x_0+x_7),\ 0,\ (x_4-x_3-x_2+x_5),\ 0, 0, 0, 0, 0]$$

$$X*Y^5 = 2^{-0.5}*[(x_2-x_1-x_0+x_3-x_4-x_5+x_6+x_7),\ 0,\ (x_4-x_1-x_2-x_3-x_0+x_5+x_6+x_7), 0, 0, 0, 0,$$

$$X*Y^6 = [(x_2+x_3-x_4-x_5),\ 0,\ (x_6-x_1-x_0+x_7),\ 0, 0, 0, 0, 0]$$

$$X*Y^7 = 2^{-0.5}*[(x_0+x_1+x_2+x_3-x_4-x_5-x_6-x_7),\ 0,\ (x_2-x_1-x_0+x_3-x_4-x_5+x_6+x_7),\ 0, 0, 0, 0,$$

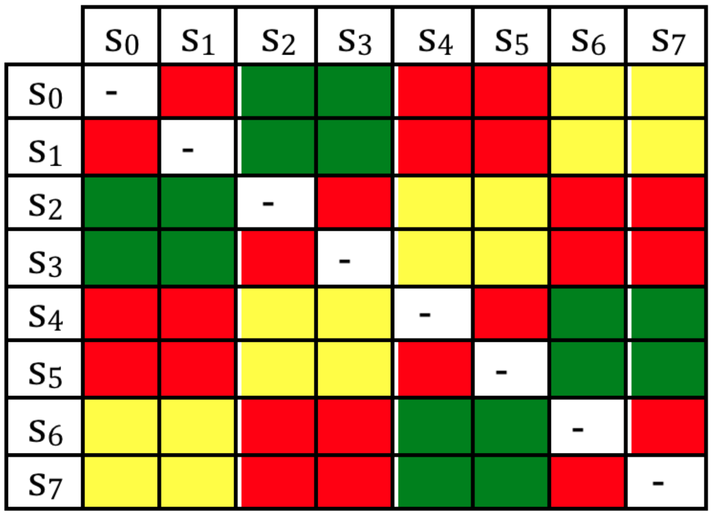
$$X*Y^8 = [(x_0+x_1-x_6-x_7),\ 0,\ (x_2+x_3-x_4-x_5),\ 0, 0, 0, 0, 0]$$

$$X*Y^9 = 2^{-0.5}*[(x_0+x_1-x_2-x_3+x_4+x_5-x_6-x_7),\ 0,\ (x_0+x_1+x_2+x_3-x_4-x_5-x_6-x_7),\ 0, 0, 0, 0,$$

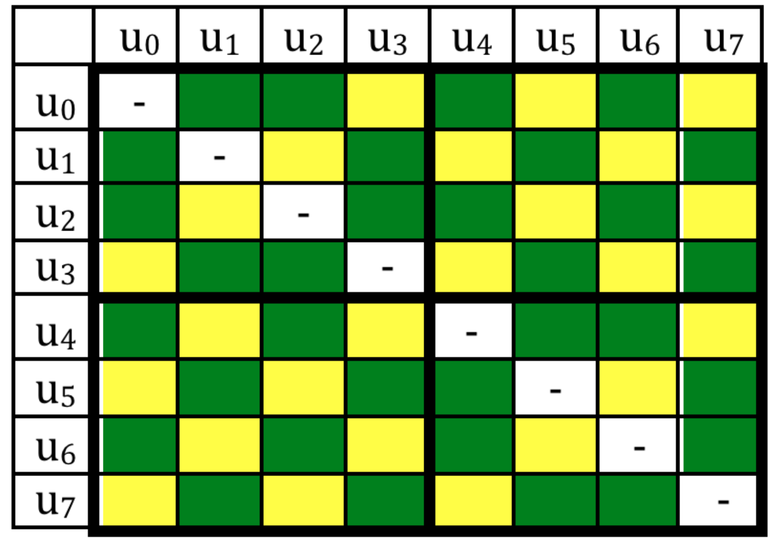
It means that this cyclic group of operators allows **a selective control (or a selective coding)** of cyclic changes of vectors in 2-dimensional plane (x_0, x_2) of a 8-dimensional vector space. Other cyclic groups, which are based on exponentiation of pairs of the genetic projectors, possess the same property of a selective control of cyclic changes in corresponding 2-dimensional planes.

Exponentiation of sums of different pairs of these genetic projectors give three kinds of results, represented in the following symmetric tables by three colors.

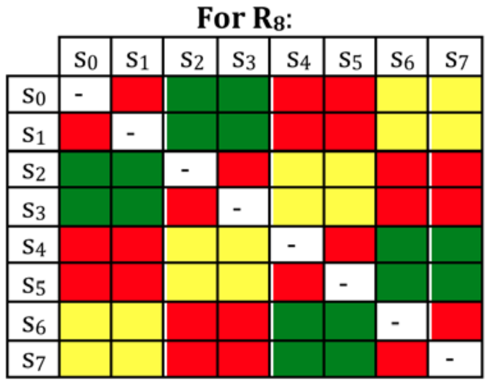
For R_8:

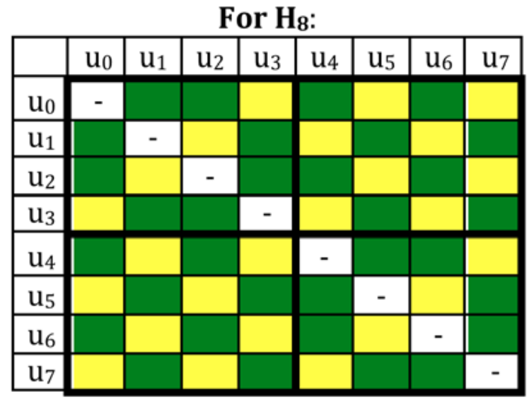


For H_8:



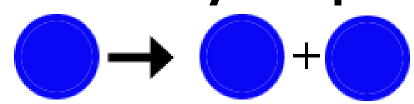
Green cells contain pairs of projectors, exponentiation of which give similar cyclic groups with the period 8. All these cyclic groups possess the property of a **selective control of cyclic changes in corresponding 2-dimensional planes** inside an 8-dimensional vector space.





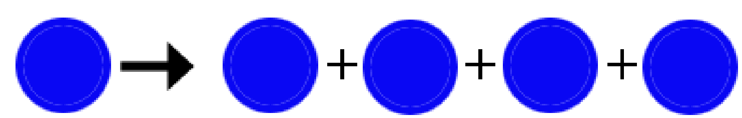
Red cells correspond to such sums of projectors, exponentiation of which shows their “doubling property” to model a dichotomous reproduction of genetic information in process of mitosis when biological cells are dichotomously reproduced:

$(s_0+s_1)^N = \mathbf{2}^{N-1}*(s_0+s_1)$, ...

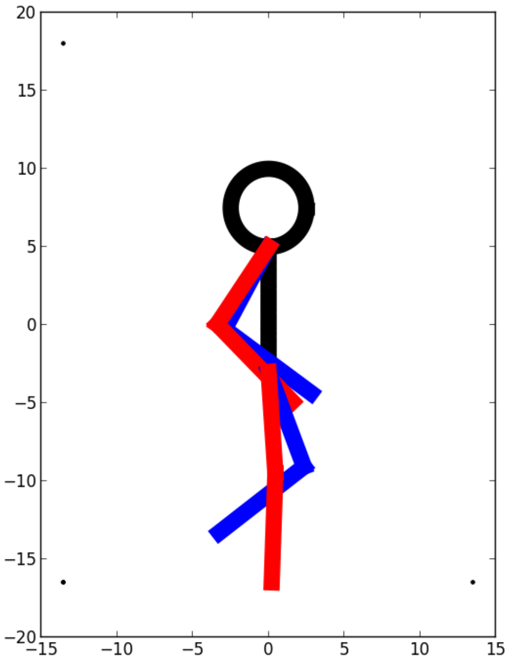


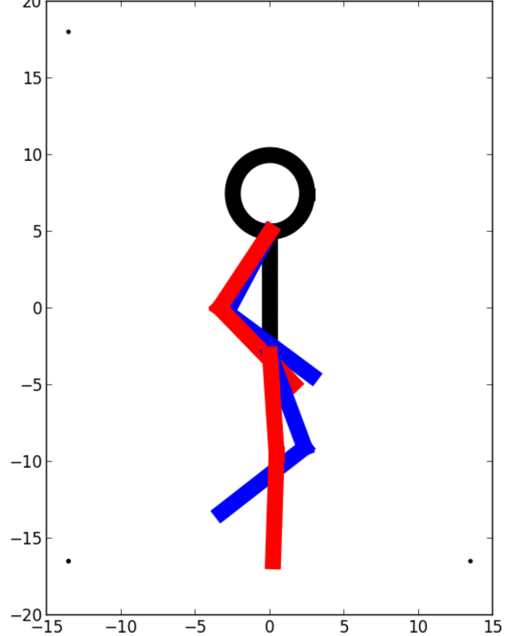
Yellow cells correspond to such sums of projectors, exponentiation of which shows their “quadruplet property” to model a quadruplet reproduction of genetic information in meiosis when gametes are quadrupletly reproduced:

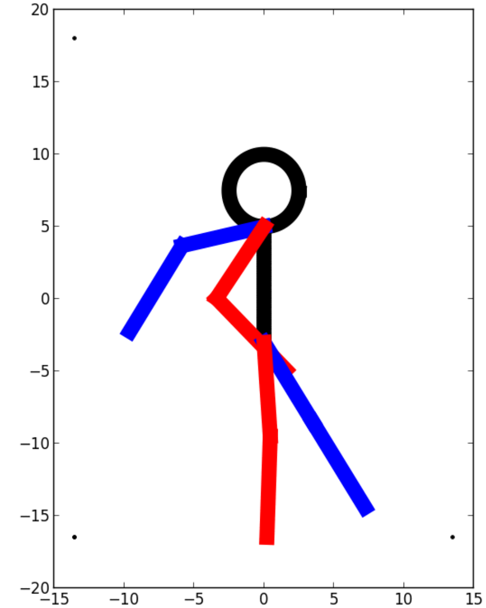
$((s_0+s_6)^2)^n = \mathbf{4}^{n-1}*(s_0+s_6)^2$, ...



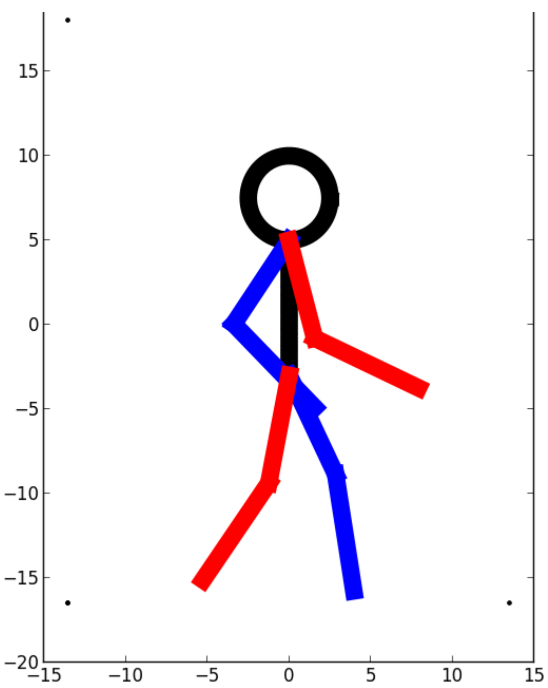
Let's return to sets of cyclic groups in green cells. The property of a simultaneously selective control of different subspaces of a multi-dimensional space by means of many cyclic groups is useful for modeling ensembles of cyclic processes in organisms including different animal gaits, etc. The simplest example is our model of human gaits, where cyclic movements of separate hands and foots can be defined independently. Fractional exponents for cyclic groups, for example $(2^{-0.5}*(s_0+s_2))^{N/K}$, allow getting any approximation to smooth (uninterrupted) movements.

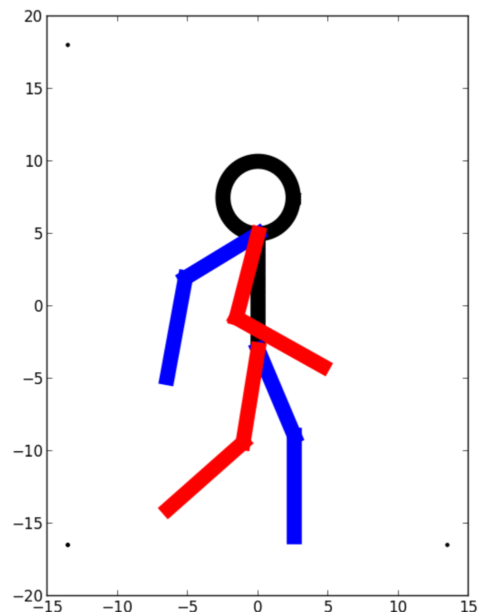


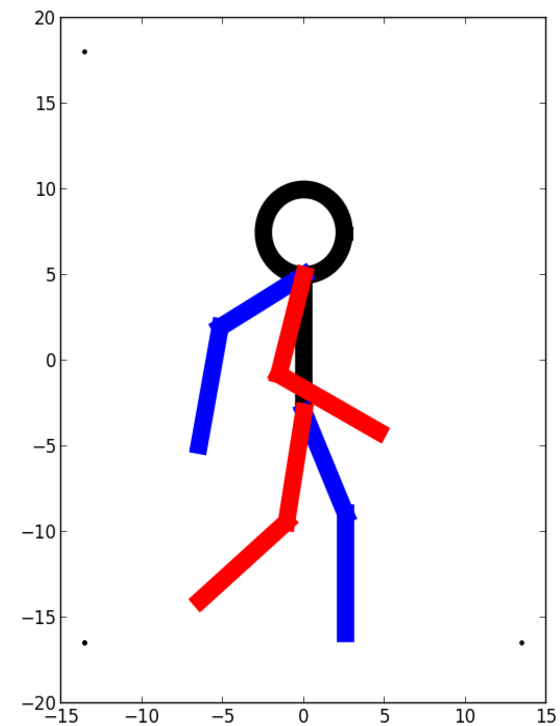




Different gaits:







But what one can do if big ensembles with thousands and more cyclic processes should be simulated?

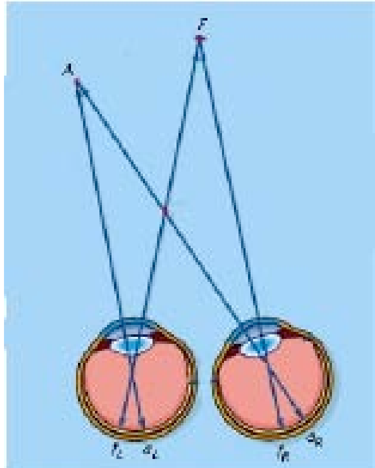
A proposed decision is based on extensions of the Rademacher's and Hadamard's (8*8)-matrices R_8 and H_8 into (2^N*2^N)-matrices by the following expressions:

$R_8 \otimes [1\ \ 1; 1\ \ 1]^{(N)}$, **$H_8 \otimes [1\ \ -1; 1\ \ 1]^{(N)}$** , where $\otimes$ means Kronecker multiplication; (N) – Kronecker power, N = 1, 2, 3, …; [1 1; 1 1] and [1 -1; 1 1] – matrix representations of complex number and double number with unit coordinates. Each of these (2^N*2^N)-matrices are sums of 2^N-projectors of the same “column type". Exponentiation of sums of different pairs of these new projectors gives as much cyclic groups as you want. These cyclic groups possess the same property of a selective control of 2-dimensional subspaces inside 2^N-dimensional space.

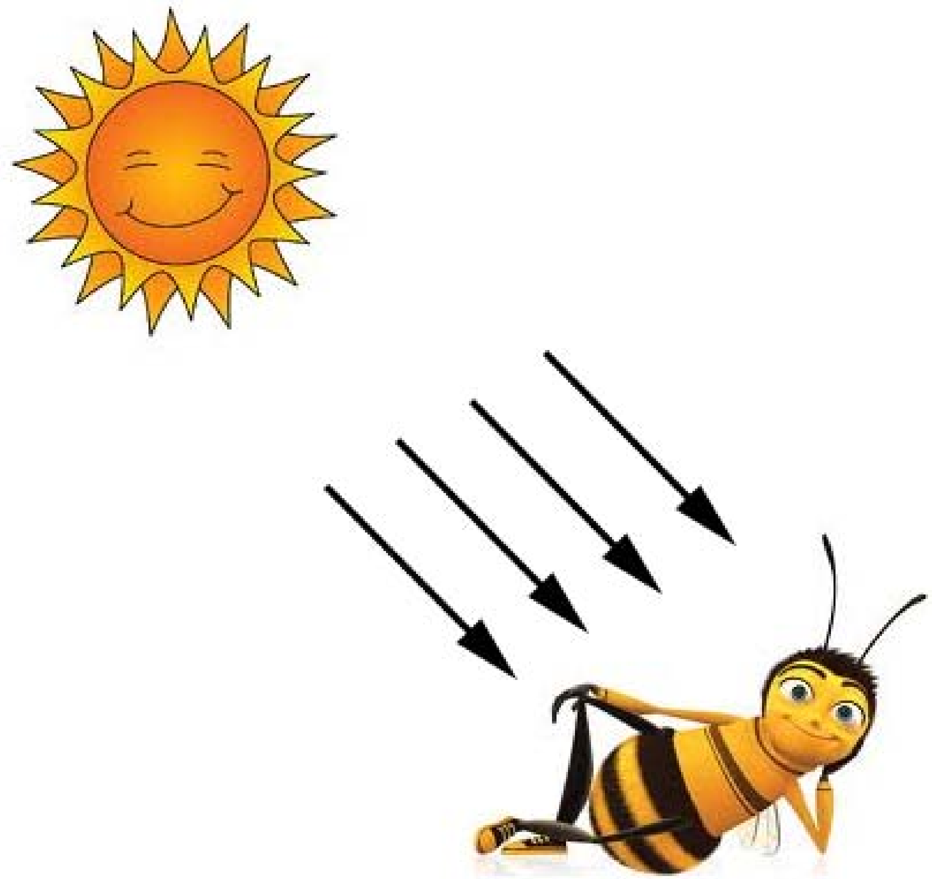
The revealed matrix approach gives new opportunities not only for studying inherited biological phenomena but also for biotechnical applications including systems of artificial intellect and robotics.

The problem of inherited ensembles of biological cycles is closely connected with a fundamental problem of biological time and biological watch.

The author puts forward a **“projectors conception”, which interprets living bodies as colonies of projection operators** and multi-dimensional constructions on a basis of direct sums of vector sub-spaces. Any organism is a whole entity, and it is naturally to think that not only visual perception is based on projectors but that all bioinformatics is connected with them.



The evolution of living organisms is connected with their absorption of **solar energy that is projected** on surfaces of biological bodies by means of solar rays. Perhaps this fact can be considered as one of reasons of importance of **projection operators** in living bodies.



ABOUT DIRECTION OF ROTATIONS

In configurations and functions of biological objects frequently one direction of rotation is preferable (it concerns the famous problem of biological dissymmetry). Taking this into account, it is interesting what one can say about a direction of rotation of vectors under influence of the cyclic groups of the considered operators? The following tables give the answer.

For R_8:

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_0	-		↺	↺				
s_1		-	↺	↺				
s_2	↺	↺	-					
s_3	↺	↺		-				
s_4					-		↺	↺
s_5						-	↺	↺
s_6					↺	↺	-	
s_7					↺	↺		-

For H_8:

	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
u_0	-	↻	↺		↻		↺	
u_1	↻	-		↺		↻		↺
u_2	↺		-	↻	↺		↺	
u_3		↺	↻	-		↺		↺
u_4	↻		↺		-	↻	↺	
u_5		↻		↺	↻	-		↺
u_6	↺		↺		↺		-	↻
u_7		↺		↺		↺	↻	-

For R_8:

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_0	-		↺	↺				
s_1		-	↺	↺				
s_2	↺	↺	-					
s_3	↺	↺		-				
s_4					-		↺	↺
s_5						-	↺	↺
s_6					↺	↺	-	
s_7					↺	↺		-

For H_8:

	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
u_0	-	↻	↺		↻		↺	
u_1	↻	-		↺		↻		↺
u_2	↺		-	↻	↺		↺	
u_3		↺	↻	-		↺		↺
u_4	↻		↺		-	↻	↺	
u_5		↻		↺	↻	-		↺
u_6	↺		↺		↺		-	↻
u_7		↺		↺		↺	↻	-

Here green cells correspond to cyclic groups on a basis of sums of pairs of projectors. The symbol ↺ means counter-clockwise rotation, the symbol ↻ means clockwise rotation. For example, the action $[x_0,x_1,x_2, x_3,x_4,x_5,x_6,x_7]*(2^{-0.5}*(s_0+s_2))^N$ gives counter-clockwise rotations of vectors in (x_0, x_2)-plane.

Tables show **dis-symetric** sets of cases of both directions of rotation: 1) the left table contains only counter-clockwise rotation; 2) the right table contains the ratio of cases ↺:↻=5:3. It generates some associations with a general problem of biological dis-symmetry.

ABOUT HAMILTON QUATERNIONS

Till now we considered sums of pairs of the genetic projectors. Now let us consider sums of 4 projectors. Hadamard matrix H_8 is sum of two sparse (8*8)-matrices $H_8 = HL_8+HR_8$, each of which is sum of 4 projectors:

$$H_8 = HL_8+HR_8 = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Each of the matrices HL_8 and HR_8 can be decomposed into 4 sparse matrices, set of which is closed in relation to multiplication and defines a known table of multiplication of Hamilton quaternions:

$HL_8 = HL_{80} + HL_{81} + HL_{82} + HL_{83} =$

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix} + \begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0&0&0&0&-1&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{bmatrix}$$

$HR_8 = HR_{80} + HR_{81} + HR_{82} + HR_{83} =$

$$\begin{bmatrix} 0&-1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&-1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0 \end{bmatrix}$$

$$\begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&-1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{bmatrix}$$

	HL_{80}	HL_{81}	HL_{82}	HL_{83}
HL_{80}	HL_{80}	HL_{81}	HL_{82}	HL_{83}
HL_{81}	HL_{81}	$-HL_{80}$	HL_{83}	$-HL_{82}$
HL_{82}	HL_{82}	$-HL_{83}$	$-HL_{80}$	HL_{81}
HL_{83}	HL_{83}	HL_{82}	$-HL_{81}$	$-HL_{80}$

	HR_{80}	HR_{81}	HR_{82}	HR_{83}
HR_{80}	HR_{80}	HR_{81}	HR_{82}	HR_{83}
HR_{81}	HR_{81}	$-HR_{80}$	HR_{83}	$-HR_{82}$
HR_{82}	HR_{82}	$-HR_{83}$	$-HR_{80}$	HR_{81}
HR_{83}	HR_{83}	HR_{82}	$-HR_{81}$	$-HR_{80}$

The multiplication table of Hamilton quaternions.

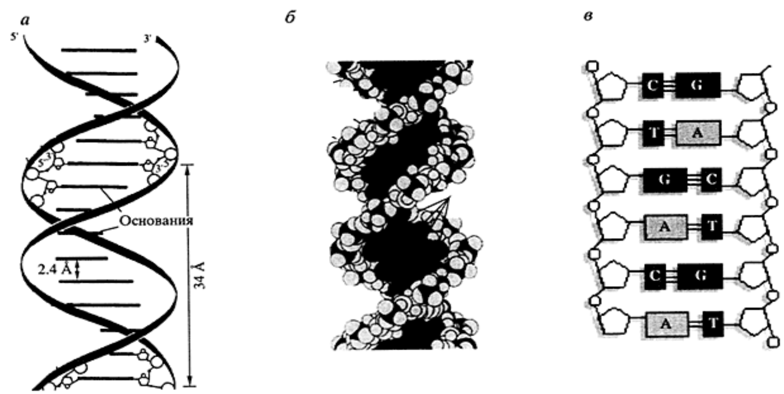
It means that the (8*8)-matrix H_8 is sum of two Hamilton quaternions with unit coordinates or, figuratively speaking, a “double quaternion”. This fact generates an association with a double helix of DNA.

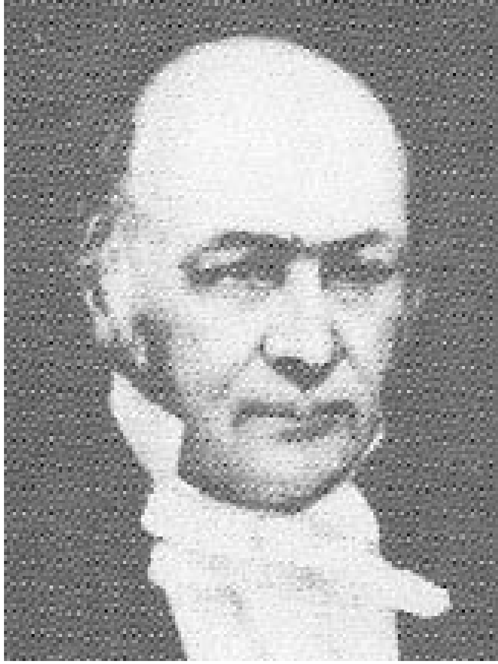
$H_8 = HL_8 + HR_8 =$

1	0	1	0	-1	0	1	0
1	0	1	0	-1	0	1	0
-1	0	1	0	1	0	1	0
-1	0	1	0	1	0	1	0
1	0	-1	0	1	0	1	0
1	0	-1	0	1	0	1	0
-1	0	-1	0	-1	0	1	0
-1	0	-1	0	-1	0	1	0

+

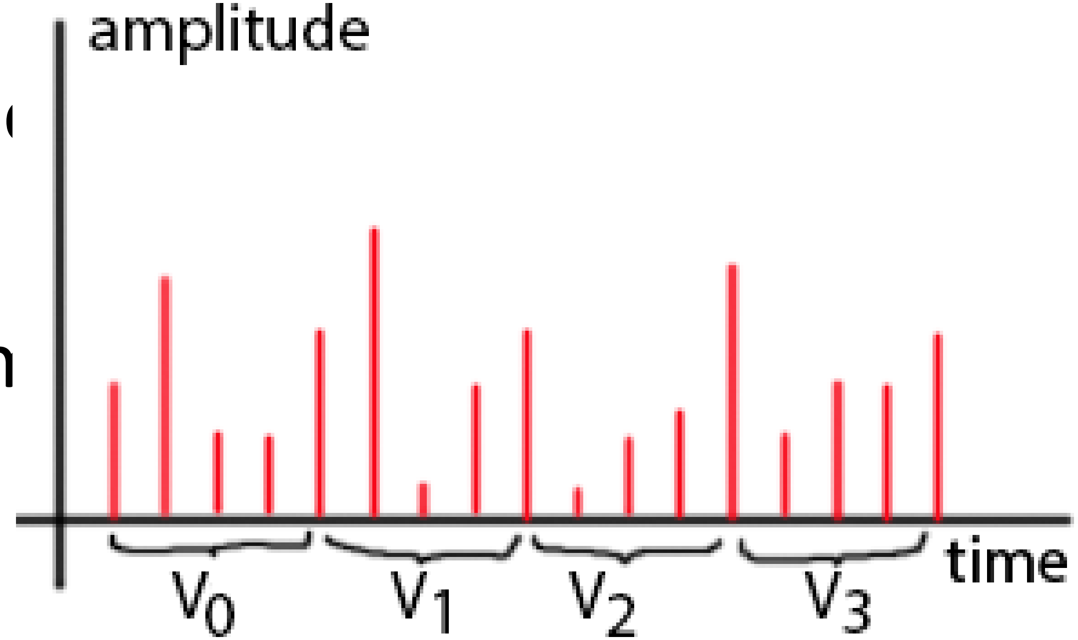
0	-1	0	-1	0	1	0	-1
0	1	0	1	0	-1	0	1
0	1	0	-1	0	-1	0	-1
0	-1	0	1	0	1	0	1
0	-1	0	1	0	-1	0	-1
0	1	0	-1	0	1	0	1
0	1	0	1	0	1	0	-1
0	-1	0	-1	0	-1	0	1





Hamilton quaternions are closely related to Pauli matrices, the theory of the electromagnetic field (Maxwell wrote his equation on the language of these quaternions), the special theory of relativity, the theory of spins, quantum theory of chemical valences, etc. In the twentieth century thousands of works were devoted to quaternions in physics [http://arxiv.org/abs/math-ph/0511092]. Now Hamilton quaternions are manifested in the genetic code system. Our scientific direction - "**matrix genetics**" - has led to the discovery of an important bridge among physics, biology and informatics for their mutual enrichment.

The connections of the genetic code with hypercomplex numbers seem to be interesting since classical **theory of noise-immunity communication is based on multi-dimensional geometry**: information sequences are represented as **sequences of multi-dimensional vectors** V_0, V_1, for the analysis of characteristics



THE 8 PROJECTORS AND HAMILTON BIQUATERNION

The genetic (8*8)-matrix H_8, which is sum of the 8 projectors, can be decomposed also in another way into a set of new 8 sparse matrices:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

$H_8 = H_{80}+H_{81}+H_{82}+H_{83}+H_{84}+H_{85}+H_{86}+H_{87} =$

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1 \end{bmatrix} + \begin{bmatrix} 0&-1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0 \end{bmatrix} + \begin{bmatrix} 0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&-1&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&-1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0 \end{bmatrix} +$$

$$\begin{bmatrix} 0&0&0&0&-1&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&-1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0 \end{bmatrix}$$

This set of 8 matrices is also closed in relation to multiplication and defines a known multiplication table of Hamilton biquaternions :

$$
\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}
+
\begin{bmatrix} 0&-1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&-1&0&0&0 \end{bmatrix}
+
$$

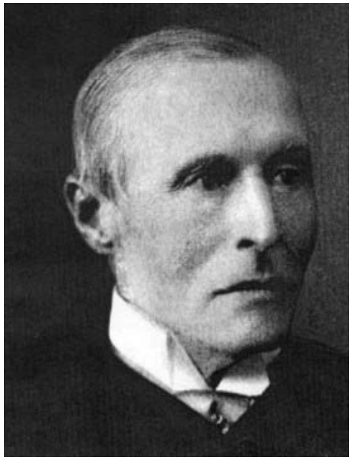
$$
\begin{bmatrix} 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0\\ 0&-1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&-1&0&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{bmatrix}
$$

	1	H_{81}	H_{82}	H_{83}	H_{84}	H_{85}	H_{86}	H_{87}
1	1	H_{81}	H_{82}	H_{83}	H_{84}	H_{85}	H_{86}	H_{87}
H_{81}	H_{81}	-1	H_{83}	$-H_{82}$	H_{85}	$-H_{84}$	H_{87}	$-H_{86}$
H_{82}	H_{82}	H_{83}	-1	$-H_{81}$	$-H_{86}$	$-H_{87}$	H_{84}	H_{85}
H_{83}	H_{83}	$-H_{82}$	$-H_{81}$	1	$-H_{87}$	H_{86}	H_{85}	$-H_{84}$
H_{84}	H_{84}	H_{85}	H_{86}	H_{87}	-1	$-H_{81}$	$-H_{82}$	$-H_{83}$
H_{85}	H_{85}	$-H_{84}$	H_{87}	$-H_{86}$	$-H_{81}$	1	$-H_{83}$	H_{82}
H_{86}	H_{86}	H_{87}	$-H_{84}$	$-H_{85}$	H_{82}	H_{83}	-1	$-H_{81}$
H_{87}	H_{87}	$-H_{86}$	$-H_{85}$	H_{84}	H_{83}	$-H_{82}$	$-H_{81}$	1

The multiplication table of Hamilton biquaternions (or Hamilton quaternions over field of complex numbers)

ABOUT SPLIT-QUATERNIONS BY J.COCKLE

The Rademacher matrix R_8 is also sum of 2 sparse (8*8)-matrices $R_8 = RL_8 + RR_8$, each of which is sum of 4 projectors:



$$R_8 = RL_8 + RR_8 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Each of the matrices RL_8 and RR_8 can be decomposed into 4 sparse matrices, set of which is closed in relation to multiplication and defines a known table of multiplication of split-quaternions by J.Cockle (1849 year, [HTTP://EN.WIKIPEDIA.ORG/WIKI/SPLIT-QUATERNION](http://en.wikipedia.org/wiki/Split-quaternion)).

RL8 =RL80+RL81+RL82+RL83=

$$
\begin{pmatrix}
1&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0
\end{pmatrix}
+
\begin{pmatrix}
0&0&1&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&-1&0&0&0\\
0&0&0&0&-1&0&0&0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&-1&0\\
1&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0\\
0&0&-1&0&0&0&0&0
\end{pmatrix}
+
\begin{pmatrix}
0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&-1&0\\
0&0&0&0&-1&0&0&0\\
0&0&0&0&-1&0&0&0\\
0&0&-1&0&0&0&0&0\\
0&0&-1&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0
\end{pmatrix}
$$

RR8 =RR80+RR81+RR82+RR83=

$$
\begin{pmatrix}
0&1&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&1
\end{pmatrix}
+
\begin{pmatrix}
0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&-1&0&0\\
0&0&0&0&0&-1&0&0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0&0&0&0&0&1&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&-1\\
0&1&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
0&0&0&-1&0&0&0&0
\end{pmatrix}
+
\begin{pmatrix}
0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&-1\\
0&0&0&0&0&-1&0&0\\
0&0&0&0&0&-1&0&0\\
0&0&0&-1&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0
\end{pmatrix}
$$

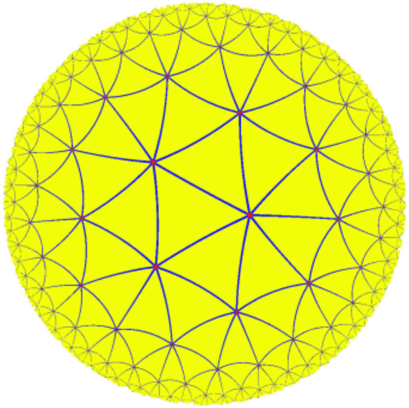
	RL_{80}	RL_{81}	RL_{82}	RL_{83}
RL_{80}	RL_{80}	RL_{81}	RL_{82}	RL_{83}
RL_{81}	RL_{81}	$-RL_{80}$	RL_{83}	$-RL_{82}$
RL_{82}	RL_{82}	$-RL_{83}$	RL_{80}	$-RL_{81}$
RL_{83}	RL_{83}	RL_{82}	RL_{81}	RL_{80}

	RR_{80}	RR_{81}	RR_{82}	RR_{83}
RR_{80}	RR_{80}	RR_{81}	RR_{82}	RR_{83}
RR_{81}	RR_{81}	$-RR_{80}$	RR_{83}	$-RR_{82}$
RR_{82}	RR_{82}	$-RR_{83}$	RR_{80}	$-RR_{81}$
RR_{83}	RR_{83}	RR_{82}	RR_{81}	RR_{80}

THE MULTIPLICATION TABLE OF SPLIT-QUATERNIONS BY J.COCKLE.

Split-quaternions by Cockle are also used in mathematics and physics, for example, in A.Poincare's model of Lobachevskiy's geometry (http://en.wikipedia.org/wiki/Split-quaternion).

	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	1	-i
k	k	j	i	1



The 8 projectors and Cockle's bi-split-quaternions

The genetic (8*8)-matrix R_8, which is sum of the 8 projectors, can be decomposed also in another way into a set of new 8 sparse matrices:

$$R_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

$R_8 = R_{80}+R_{81}+R_{82}+R_{83}+R_{84}+R_{85}+R_{86}+R_{87} =$

$$\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix} + \begin{bmatrix} 0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix} + \begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&-1&0&0&0 \\ 0&0&0&0&0&-1&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&-1&0&0 \\ 0&0&0&0&-1&0&0&0 \end{bmatrix} +$$

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&-1&0 \\ 0&0&0&0&0&0&0&-1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&0&0&-1&0 \\ 0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&-1&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&-1&0&0&0 \\ 0&0&0&0&0&-1&0&0 \\ 0&0&-1&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&0&0&-1&0 \\ 0&0&0&0&0&-1&0&0 \\ 0&0&0&0&-1&0&0&0 \\ 0&0&0&-1&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \end{bmatrix}$$

This set of 8 matrices are also closed in relation to multiplication and defines a known multiplication table of Cockle's biquaternions :

$$
\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}
+
\begin{bmatrix} 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&-1&0&0&0 \end{bmatrix}
+
$$

$$
\begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&-1&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&-1&0&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{bmatrix}
+
\begin{bmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{bmatrix}
$$

	$R0_8$	$R1_8$	$R2_8$	$R3_8$	$R4_8$	$R5_8$	$R6_8$	$R7_8$
$R0_8$	$R0_8$	$R1_8$	$R2_8$	$R3_8$	$R4_8$	$R5_8$	$R6_8$	$R7_8$
$R1_8$	$R1_8$	$R0_8$	$R3_8$	$R2_8$	$R5_8$	$R4_8$	$R7_8$	$R6_8$
$R2_8$	$R2_8$	$R3_8$	$-R0_8$	$-R1_8$	$R6_8$	$R7_8$	$-R4_8$	$-R5_8$
$R3_8$	$R3_8$	$R2_8$	$-R1_8$	$-R0_8$	$R7_8$	$R6_8$	$-R5_8$	$-R4_8$
$R4_8$	$R4_8$	$R5_8$	$-R6_8$	$-R7_8$	$R0_8$	$R1_8$	$-R2_8$	$-R3_8$
$R5_8$	$R5_8$	$R4_8$	$-R7_8$	$-R6_8$	$R1_8$	$R0_8$	$-R3_8$	$-R2_8$
$R6_8$	$R6_8$	$R7_8$	$R4_8$	$R5_8$	$R2_8$	$R3_8$	$R0_8$	$R1_8$
$R7_8$	$R7_8$	$R6_8$	$R5_8$	$R4_8$	$R3_8$	$R2_8$	$R1_8$	$R0_8$

The multiplication table of bisplit-quaternion by J.Cockle (or split-quaternions over field of complex numbers)

Here we have received new examples of the effectiveness of mathematics: abstract mathematical structures, which have been derived by mathematicians at the tip of the pen 160 years ago, are embodied long ago in the information basis of living matter - the system of genetic coding. The mathematical structures, which are discovered by mathematicians in a result of painful reflections (like Hamilton, who has wasted 10 years of continuous thought to reveal his quaternions), are already represented in the genetic coding system.

Let’s return to (8*8)-matrix representations **HL_8** and **HR_8** of Hamilton quaternions with unit coordinates.

$$HL_8 = \begin{array}{|c|c|c|c|c|c|c|c|}\hline 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ \hline -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ \hline -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ \hline -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ \hline \end{array} \; ; \quad HR_8 = \begin{array}{|c|c|c|c|c|c|c|c|}\hline 0 & -1 & 0 & -1 & 0 & 1 & 0 & -1 \\ \hline 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ \hline 0 & 1 & 0 & -1 & 0 & -1 & 0 & -1 \\ \hline 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \hline 0 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \\ \hline 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ \hline 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \\ \hline \end{array}$$

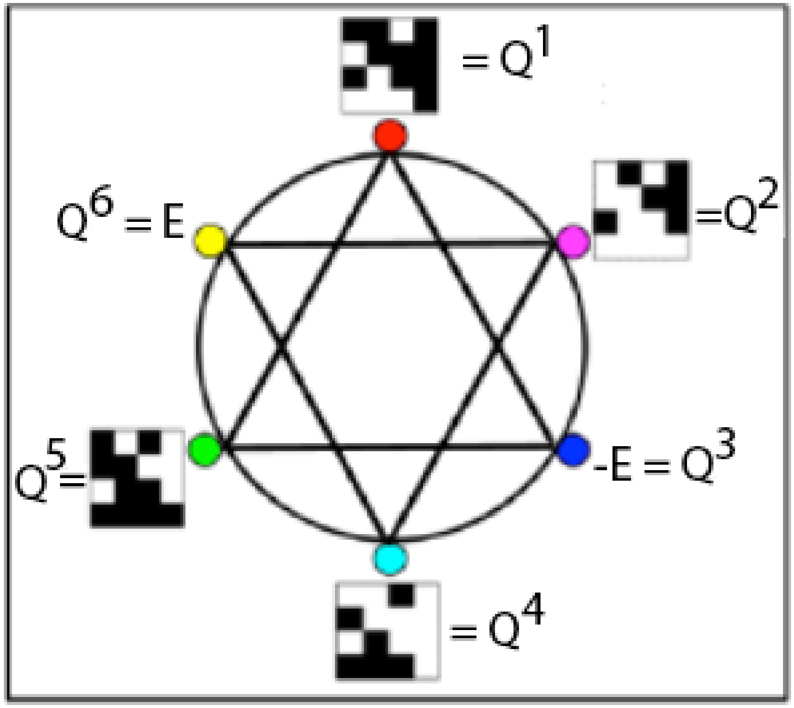
Exponentiation of each of these matrices (with a coefficient 0.5) leads to a cyclic group with its period **6** (n = 1, 2, 3, …): $\mathbf{(0.5*HL_8)^{n+6} = (0.5*HL_8)^{n}}$; $\mathbf{(0.5*HR_8)^{n+6} = (0.5*HR_8)^{n}}$.

A similar expression is true for a classical (4*4)-matrix representation of Hamilton quaternion **Q** with unit coordinates: $\mathbf{(0.5*Q)^{n+6} = (0.5*Q)^{n}}$

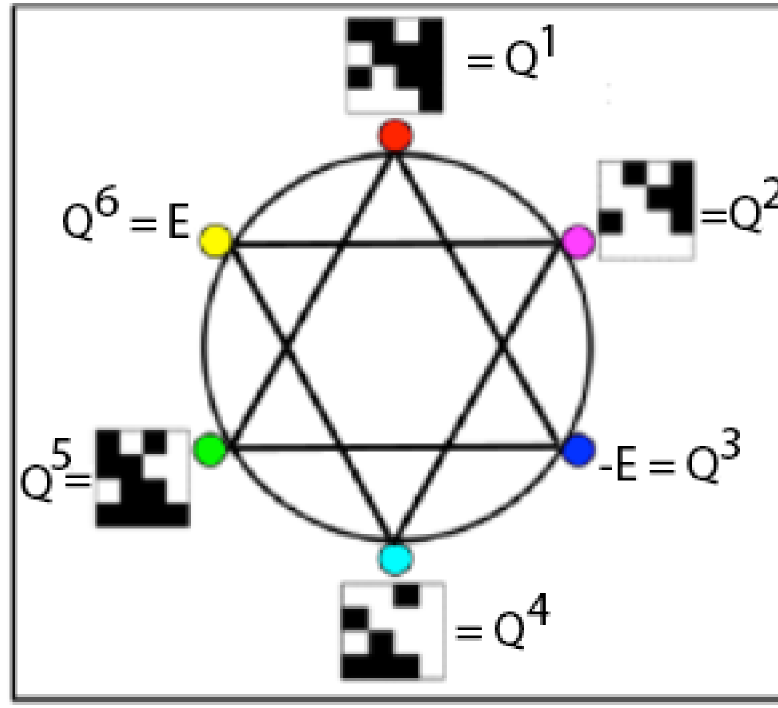
$$Q = 0.5* \begin{array}{|c|c|c|c|}\hline 1 & 1 & -1 & 1 \\ \hline -1 & 1 & 1 & 1 \\ \hline 1 & -1 & 1 & 1 \\ \hline -1 & -1 & -1 & 1 \\ \hline \end{array}$$

One can dispose all 6 members of any of these cyclic groups (for example, members of the group of (4*4)-matrices **$(0.5*Q)^{n+6}$ =$(0.5*Q)^{n}$**) on a circle to show a complete analogy of their set to famous **Newton's color circle of inborn properties of human color perception**.

$$Q = 0.5* \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$



The Newton's color circle shows the following:
1) each of 6 colors on the circle is the sum of two adjacent colors (the same is true for these 6 quaternions on the circle);
2) the three colors in vertices of each of 2 triangles of the "star of David" neutralize each other in their summation (the same is true for the quaternions in each of triangle of the "star of David", whose sum is equal to 0).
3) complementary colors, which are opposite each other on this circle, neutralize each other in their summation (sum of any two diagonal quaternions on the circle is also equal to 0).



Briefly speaking, the red, magenta, blue, cyan, green and yellow colors are formally expressed by means of the Hamilton quaternions Q^1, Q^2, Q^3, Q^4, Q^5, Q^6 correspondingly. The problem of mixing of colors can now be solved in terms of the cyclic group of Hamilton quaternions Q^n.

Using the mentioned (2^n*2^n)-matrix representations of Hamilton quaternions allows encoding (or controlling) different colors in different sub-spaces of an internal space of a living body.

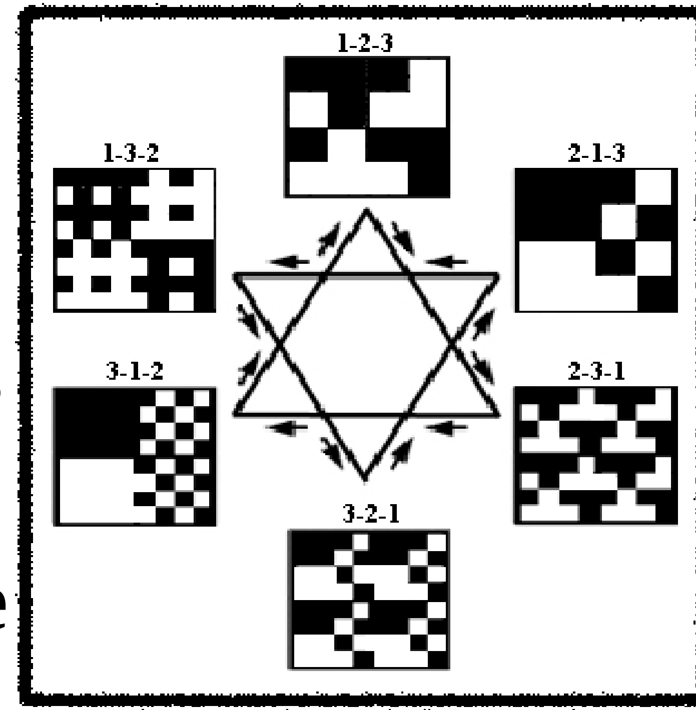
Algebraic invariances and positional permutations in triplets

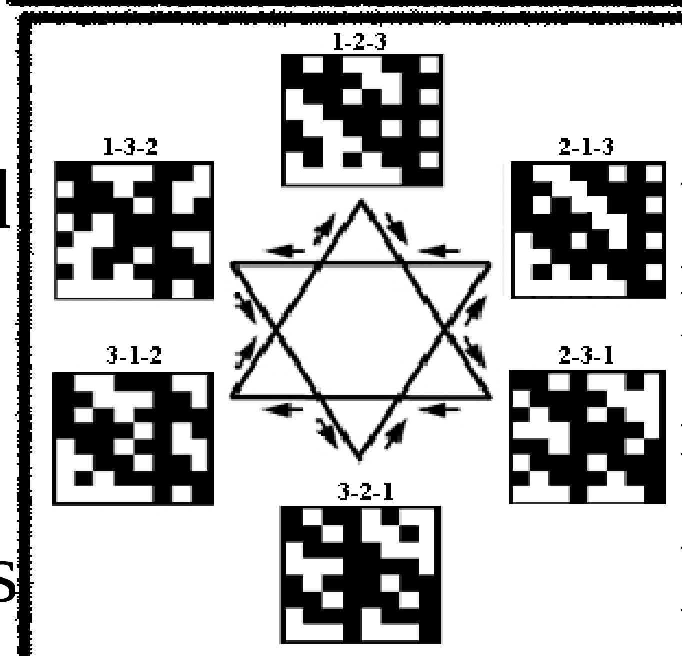
The theory of signal processing pays a special attention to permutations of information elements. Six variants of permutations of positions inside a triplet exist: 1-2-3, 2-3-1, 3-1-2, 3-2-1, 2-1-3, 1-3-2, 3-2-1.

Let us study transformations of the Rademacher and Hadamard representations of the genomatrix [C T; A G]$^{(3)}$ in all these cases of positional permutations in all triplets. A simultaneous permutation of positions in triplets transforms the most of the triplets in its matrix cells. For example, in the case of the transformation of the order of positions 1-2-3 into the order 2-3-1, the "white" triplet **CA**G with the weak root CA is transformed into the "black" triplet **AG**C with the strong root AG. In the result, **the quite new mosaic genomatrices arises**.

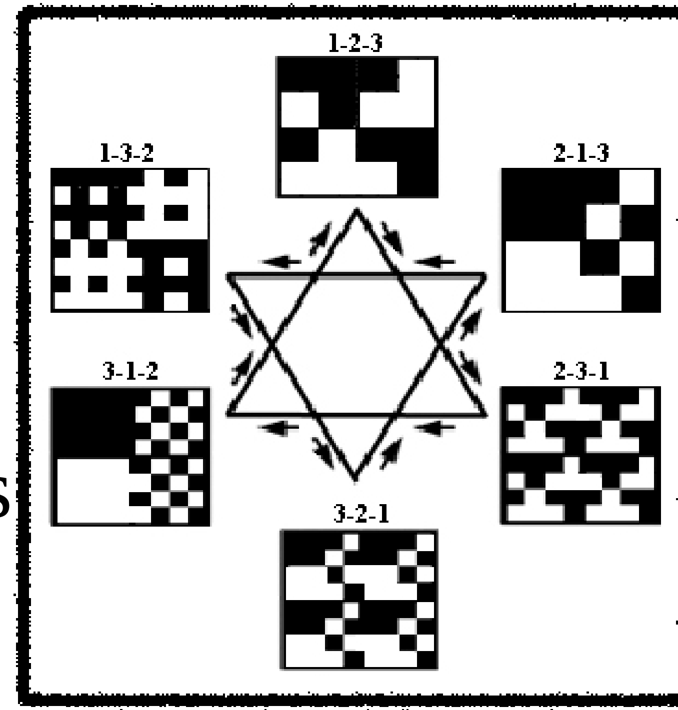
In the result of these positional permutations in triplets, **five additional Rademacher matrices** arise from the initial Rademacher genomatrix R_8; each of them is a **new matrix representation of the same bi-spit-quaternion by Cockle** with unit coordinates.

Also in the result of these permutations, **five new Hadamard matrices** come from the initial Hadamard genomatrix H_8; each of them is **a new representation of the same Hamilton's biquaternion** with unit coordinates

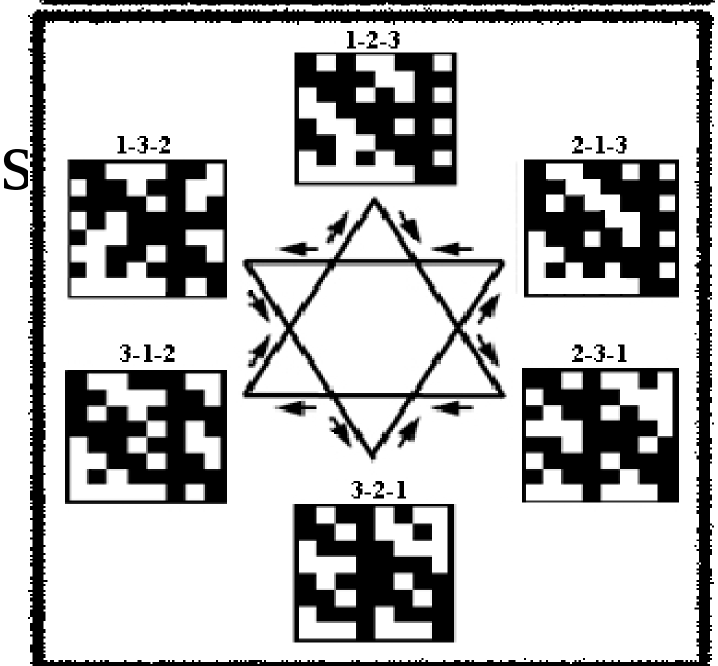




Each of these 5 new Rademacher matrices can be also decomposed into 8 projectors with Rademacher functions in their non-zero columns.

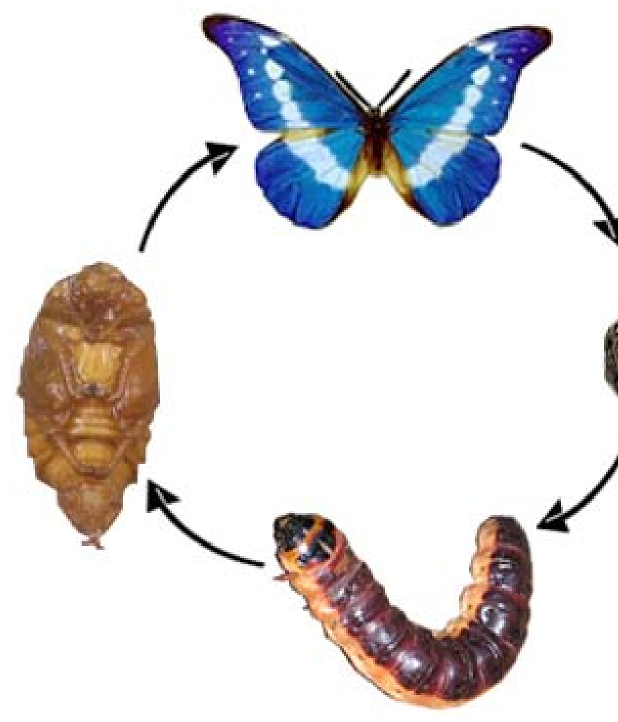


Each of these 5 new Hadamard matrices can be also decomposed into 8 projectors with Walsh functions in their non-zero columns.



All ideology of projectors and their combinations is conserved for these new matrices, which arise due to positional permutations in triplets.

The invariance of matrix algebras with different permutations of elements in genomatrices is interesting, in particular, due to the metamorphosis of the organisms. For example, in the metamorphosis of a butterfly, chrysalis does not eat at all and has a fixed atomic composition, but - by means of genetically determined permutations of elements – chrysalis turns into a butterfly, which is a quite different organism with the same DNA.



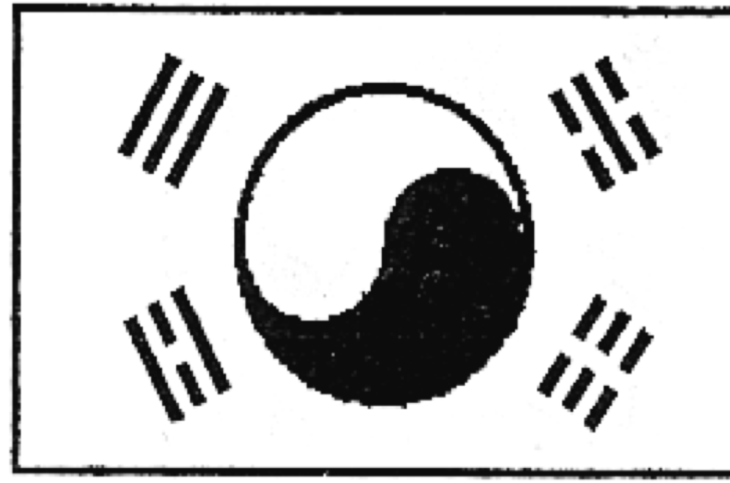
It seems that the nature likes projectors. For example, electromagnetic vectors are represented as sums of their projections in a form of electric and magnetic vectors.

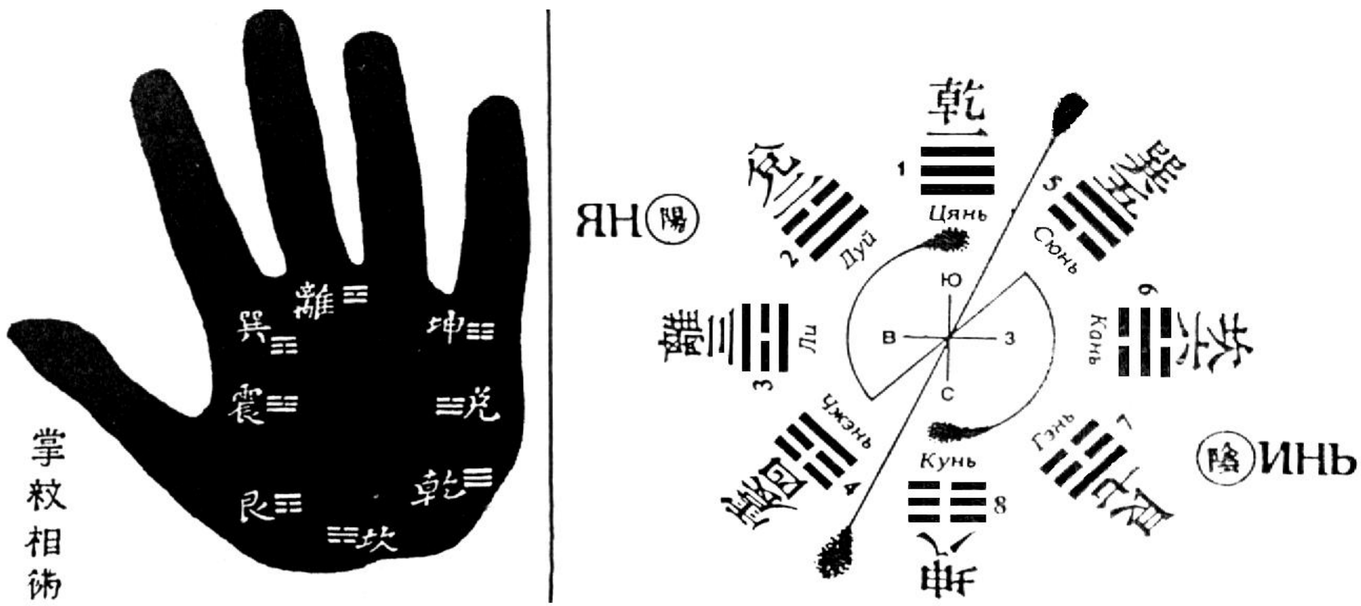
CYCLIC CHANGES AND THE "I-CHING"

In the field of molecular genetics, Nobel prize winner F.Jacob, a famous Prof. G.Stent (1965) and some other authors already noted some parallelisms between the molecular genetic system and a symbolic system of the Ancient Chinese book "I-Ching" ("The Book of Changes"), which was written a few thousand years ago.

This book had a great influences on different aspects of life of people not only in China but also in many other countries.

The state flag of South Korea with symbols of triplets from "I-Ching"





Cyclic and other patterns, which arise in “matrix genetics”, have many new analogies with the system of “I-Ching”, Chinese circular calendars, the Zodiac system and patterns of Ancient Oriental medicine. In other words, here we get new materials for a problem of “a connection of times”.


ЗМЕЯ
ЛОШАДЬ
КОЗА (ОВЦА)
ОБЕЗЬЯНА
ПЕТУХ
СОБАКА
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КРЫСА
БЫК
ТИГР
КРОЛИК
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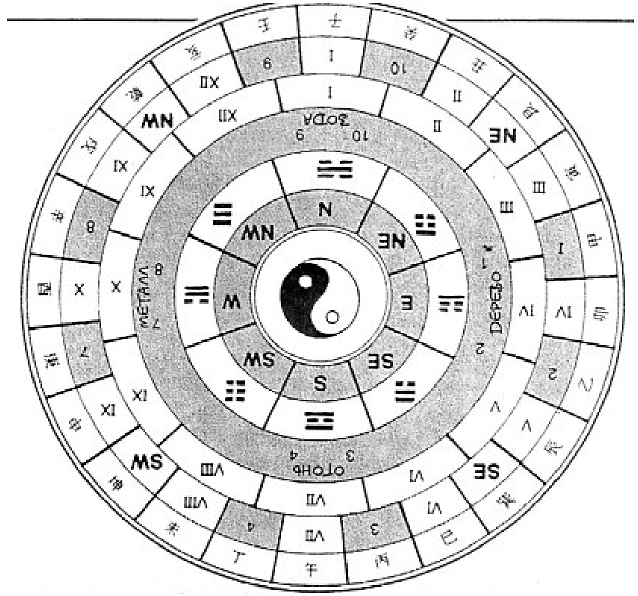
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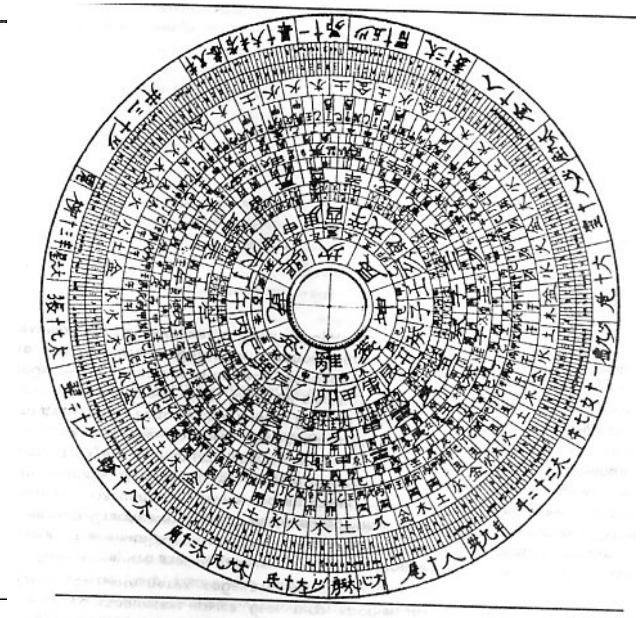
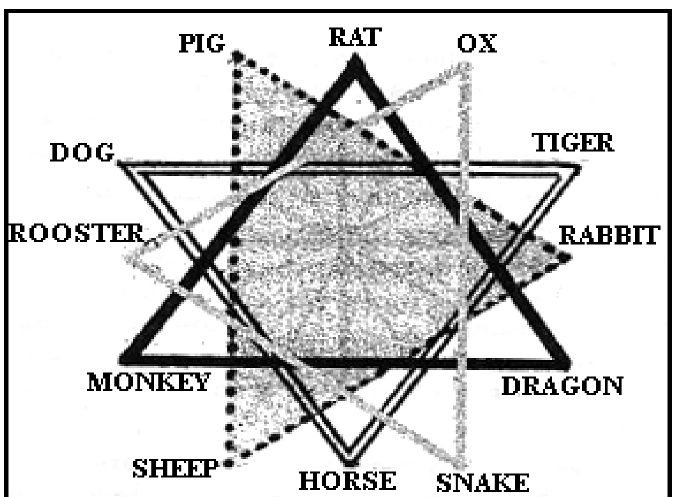
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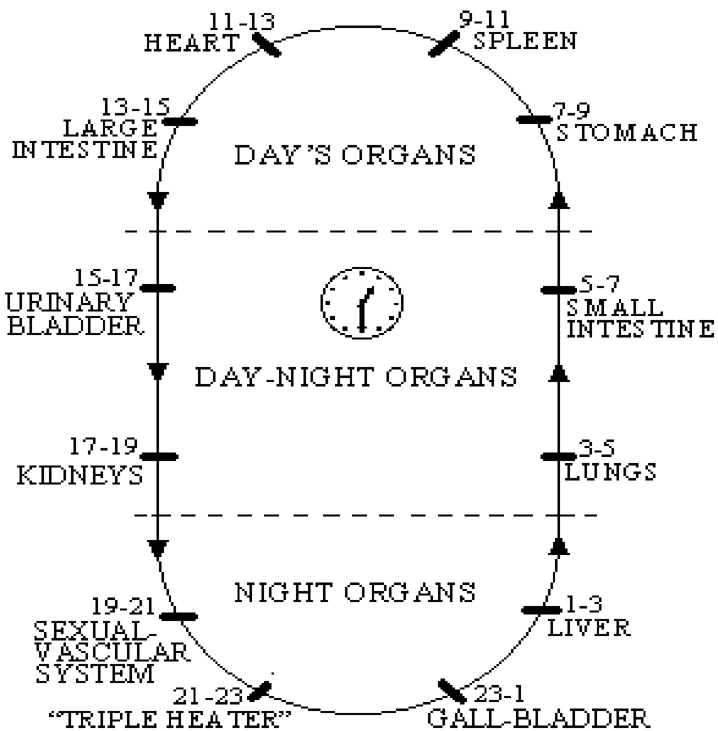
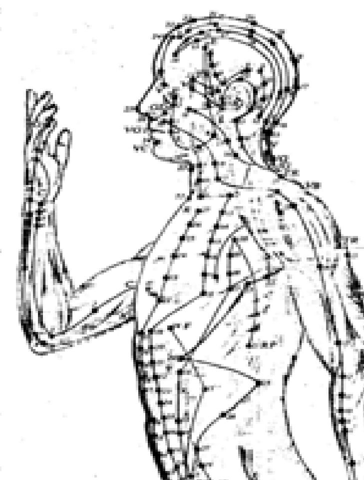
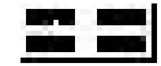
RAT
OX
DOG
TIGER
ROOSTER
RABBIT
MONKEY
DRAGON
SHEEP
HORSE
SNAKE

11-13 HEART
9-11 SPLEEN
13-15 LARGE INTESTINE
7-9 STOMACH
DAY'S ORGANS
15-17 URINARY BLADDER
5-7 SMALL INTESTINE
DAY-NIGHT ORGANS
17-19 KIDNEYS
3-5 LUNGS
NIGHT ORGANS
19-21 SEXUAL-VASCULAR SYSTEM
1-3 LIVER
21-23 "TRIPLE HEATER"
23-1 GALL-BLADDER


“I-Ching” deals with Yin-Yang symbols including the four basic digrams: Old Yang (), Old Yin (), Young Yang () and Young Yin ().

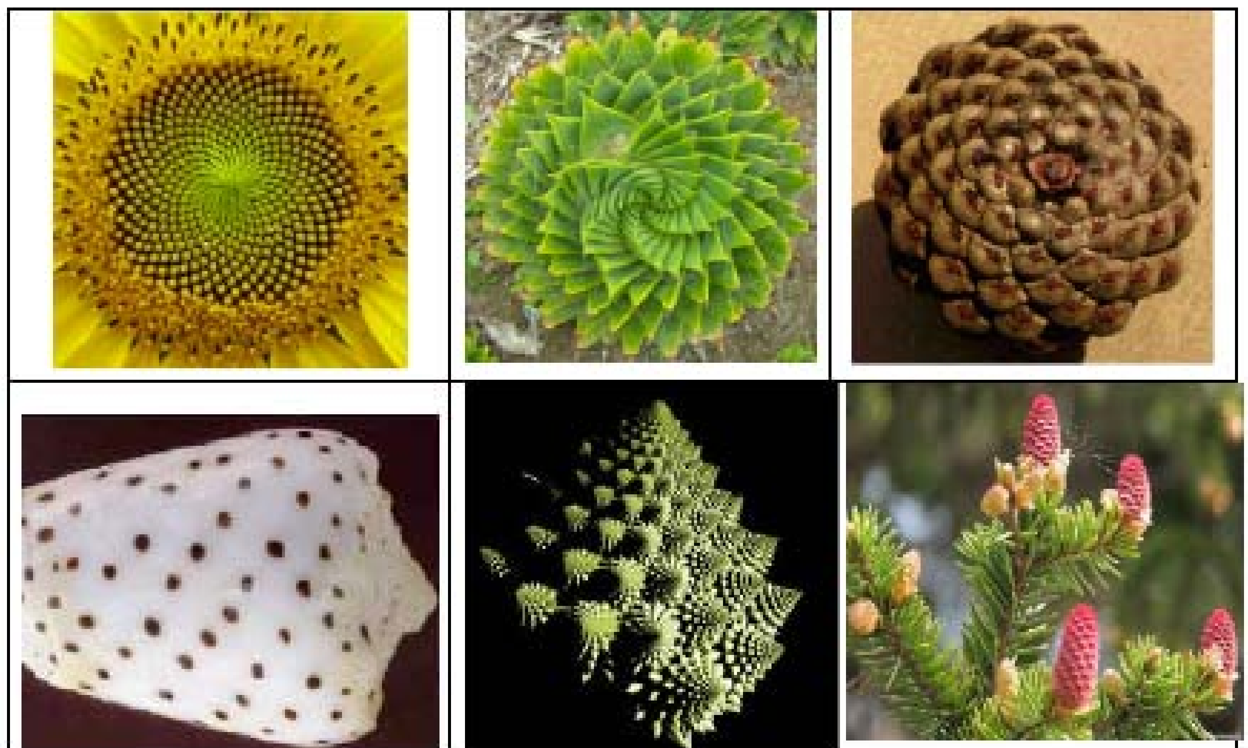
The famous table of 64 hexagrams in Fu-Xi’s order exists in this symbolic system:

	111 CHYAN	110 TUI	101 LI	100 CHEN	011 HSUN	010 KAN	001 KEN	000 KUN
111 CHYAN	*111*111	*111*110	*111*101	*111*100	*111*011	*111*010	*111*001	*111*000
110 TUI	*110*111	*110*110	*110*101	*110*100	*110*011	*110*010	*110*001	*110*000
101 LI	*101*111	*101*110	*101*101	*101*100	*101*011	*101*010	*101*001	*101*000
100 CHEN	*100*111	*100*110	*100*101	*100*100	*100*011	*100*010	*100*001	*100*000
011 HSUN	*011*111	*011*110	*011*101	*011*100	*011*011	*011*010	*011*001	*011*000
010 KAN	*010*111	*010*110	*010*101	*010*100	*010*011	*010*010	*010*001	*010*000
001 KEN	*001*111	*001*110	*001*101	*001*100	*001*011	*001*010	*001*001	*001*000
000 KUN	*000*111	*000*110	*000*101	*000*100	*000*011	*000*010	*000*001	*000*000

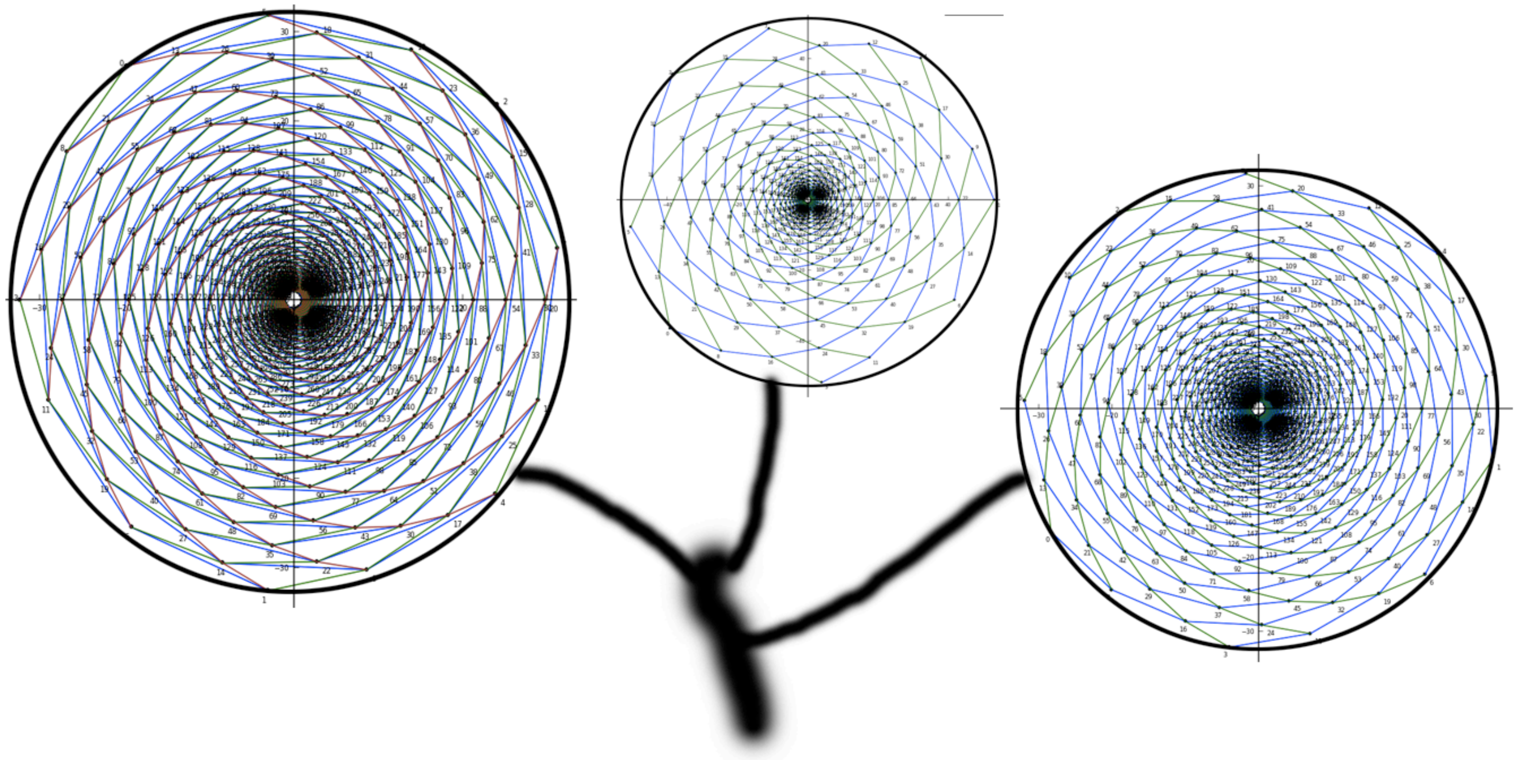
The ancient Chinese claimed that the system of ”I-Ching" is a universal archetype of nature, a universal classification system. They knew nothing about the genetic code, but the genetic code is constructed in accordance with the ”I-Ching".

Briefly about ensembles of phyllotaxis patterns

Let us briefly note that a study of sums of genetic projectors has given new possibilities of modeling some other inherited biological phenomena including a phenomenon of ensembles of phyllotaxis patterns inside one organism.



It is known that an organism can have many phyllotaxis patterns in its parts. Figure shows an example of a spruce with many phyllotaxis cones. Each of these cones can be interpreted as a sub-space of a multi-dimensional internal space of this tree. The proposed approach of genetic projectors and their sums allows modeling such ensemble of phyllotaxis patterns, each of which is realized in its own subspace with an individual velocity and a phase shift of development (see some details in the article [Petoukhov, 2013, http://arxiv.org/abs/1307.7882]).



An example of modeling three phyllotaxis patterns, each of which belongs to its own 2-dimensional sub-space of a general multi-dimensional internal space.

PROJECTORS AND THE EXCLUSION PRINCILE OF EVOLUTION OF DIALECTS OF THE GENETIC CODE

Science knows 19 dialects of the genetic code http://www.ncbi.nlm.nih.gov/Taxonomy/Utils/wprintgc.cgi . Some of them have another black-and-white mosaic in their matrix presentation [C U; A G]$^{(3)}$ (another system of triplets with strong and weak roots).

The Standard Code
The Vertebrate Mitochondrial Code
The Yeast Mitochondrial Code
The Mold, Protozoan, and Coelenterate Mitochondrial Code and
Mycoplasma/Spiroplasma Code
The Invertebrate Mitochondrial Code
The Ciliate, Dasycladacean and Hexamita Nuclear Code
The Echinoderm and Flatworm Mitochondrial Code
The Euplotid Nuclear Code
The Bacterial, Archaeal and Plant Plastid Code
The Alternative Yeast Nuclear Code
The Ascidian Mitochondrial Code
The Alternative Flatworm Mitochondrial Code
Blepharisma Nuclear Code
Chlorophycean Mitochondrial Code
Trematode Mitochondrial Code
Scenedesmus Obliquus Mitochondrial Code
Thraustochytrium Mitochondrial Code
Pterobranchia Mitochondrial Code
Candidate Division SR1 and Gracilibacteria Code

Some of the dialects have another black-and-white mosaic in their matrix presentation [C U; A G](3) (another system of triplets with strong and weak

The Vertebrate Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Met	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Stop	AGG Stop	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Standard Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Invertebrate Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Met	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Ser	AGG Ser	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Echinoderm and Flatworm Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Asn	AAG Lys	AGA Ser	AGG Ser	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Yeast Mitochondrial Code:

CCC Pro	CCU Pro	CUC Thr	CUA Thr	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUU Thr	CUG Thr	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Met	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Euplotid Nuclear Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Cys	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Bacterial, Archaeal and Plant Plastid Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Alternative Yeast Nuclear Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Ser	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Ascidian Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Met	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Gly	AGG Gly	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Alternative Flatworm Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Asn	AAG Lys	AGA Ser	AGG Ser	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Blepharisma Nuclear Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Gln	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Chlorophycean Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Leu	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Trematode Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Met	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Asn	AAG Lys	AGA Ser	AGG Ser	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Scenedesmus Obliquus Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Stop	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Leu	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Thraustochytrium Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Stop	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Pterobranchia Mitochondrial Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Ser	AGG Lys	GAA Glu	GAG Glu	GGA Gly	GGG Gly

Candidate Division SR1 and Gracilibacteria Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Gly	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Mold, Protozoan, and Coelenterate Mitochondrial Code and the Mycoplasma/Spiroplasma Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Ciliate, Dasycladacean and Hexamita Nuclear Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Gln	UAG Gln	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The set of 19 dialects of the genetic code contains 13 dialects with a “typical mosaic” of their matrix representations and 6 dialects with non-typical mosai

Candidate Division SR1 and Gracilibacteria Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Gly	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Mold, Protozoan, and Coelenterate Mitochondrial Code and the Mycoplasma/Spiroplasma Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Stop	UAG Stop	UGA Trp	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The Ciliate, Dasycladacean and Hexamita Nuclear Code:

CCC Pro	CCU Pro	CUC Leu	CUU Leu	UCC Ser	UCU Ser	UUC Phe	UUU Phe
CCA Pro	CCG Pro	CUA Leu	CUG Leu	UCA Ser	UCG Ser	UUA Leu	UUG Leu
CAC His	CAU His	CGC Arg	CGU Arg	UAC Tyr	UAU Tyr	UGC Cys	UGU Cys
CAA Gln	CAG Gln	CGA Arg	CGG Arg	UAA Gln	UAG Gln	UGA Stop	UGG trp
ACC Thr	ACU Thr	AUC Ile	AUU Ile	GCC Ala	GCU Ala	GUC Val	GUU Val
ACA Thr	ACG Thr	AUA Ile	AUG Met	GCA Ala	GCG Ala	GUA Val	GUG Val
AAC Asn	AAU Asn	AGC Ser	AGU Ser	GAC Asp	GAU Asp	GGC Gly	GGU Gly
AAA Lys	AAG Lys	AGA Arg	AGG Arg	GAA Glu	GAG Glu	GGA Gly	GGG Gly

The set of 19 dialects of the genetic code contains 13 dialects with a "typical mosaic" of their matrix representations and 6 dialects with non-typical mosaics.

The author notes the following non-trivial phenomenological fact: If each of black (white) triplets is replaced by +1(-1) in the matrices, **every of these numeric matrices of the genetic dialects is sum of projector operators**. One can name this fact as the "**projection rule**" of dialects of the genetic code.

So the following **exclusion principle is proposed**:

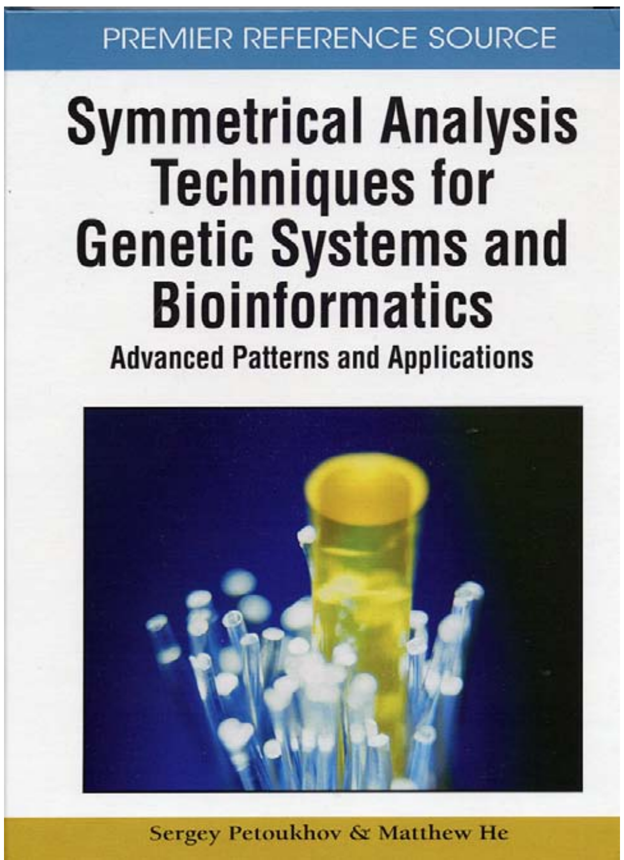
- **nature forbids such dialects of the genetic code, in which the division of the set of 64 triplets into two subsets of triplets with strong and weak roots leads to a violation of "the projection rule"**.

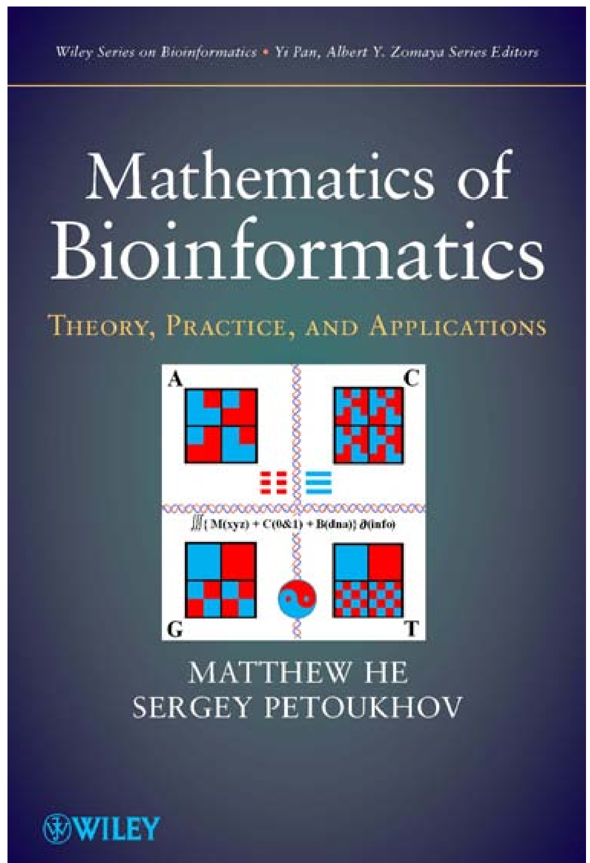
Discovering exclusive principles of nature is an important task of mathematical natural science (the exclusive principle by Pauli in quantum mechanics is one of examples).

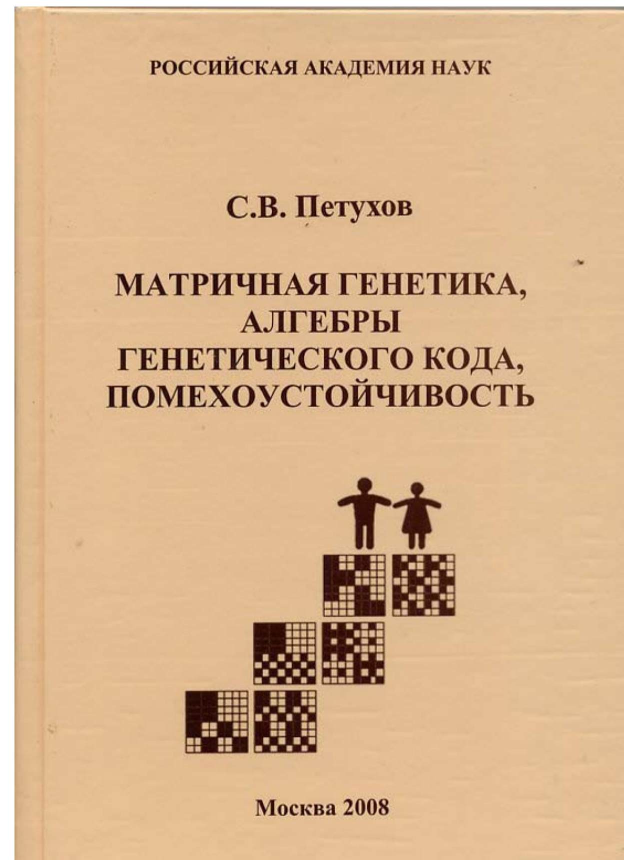
About the main role of informatics in living matter:

"***Notions of "information" or "valuable information" are not utilized in physics of non-biological nature because they are not needed there. On the contrary, <u>in biology notions "information" and especially "valuable information" are main ones</u>; understanding and description of phenomena in biological nature are <u>impossible without these notions. A specificity of "living substance" lies in these notions</u>***" (Chernavskiy, 2000, "The problem of origin of life and thought from the viewpoint of the modern physics",- "*Progress of Physical Sciences*", 170(2), p.157-183 ("*Uspehi Physicheskih Nauk*", in Russian)). Prof. Chernavskiy is a head of Department of theoretical biophysics in Physical Institute of the Russian Academy of Sciences.

The author has published four books about matrix genetics in Russia (2001, 2008) and in the USA (2010 and 2011 years) and many thematic articles (see his personal web site http://petoukhov.com/).







But materials of this lecture are presented mainly in the article:

S.Petoukhov “The genetic code, algebra of projection operators and problems of inherited biological ensembles ” http://arxiv.org/abs/1307.7882

CONCLUSIONS

Projection operators are one of the most useful notions and mathematical instruments to study the genetic coding system and genetically inherited biological phenomena including ensembles of cyclic processes. Living matter is an algebraic essence in its informational fundamentals. A development of algebraic biology is possible with using approaches of matrix genetics.

THANK YOU FOR YOUR ATTENTION!

S.Petoukhov: http://petoukhov.com/ , http://symmetry.hu/isabm/petoukhov.html

Acknowledgments. Described researches were made by the author in a frame of a long-term cooperation between Russian and Hungarian Academies of Sciences. The author is grateful to Darvas G., Stepanyan I.V., Svirin V.I. for their collaboration.

Fractal genetic networks and rules of long nucleotide sequences are presented in the article:

“Fractal genetic nets and symmetry principles in long nucleotide sequences” - “Symmetries in genetic information and algebraic biology”, специальный выпуск журнала “Symmetry: Culture and Science”, Guest editor: S. Petoukhov, 2012, vol. 23, № 3-4, p. 303-322. http://symmetry.hu/scs_online/SCS_23_3-4.pdf

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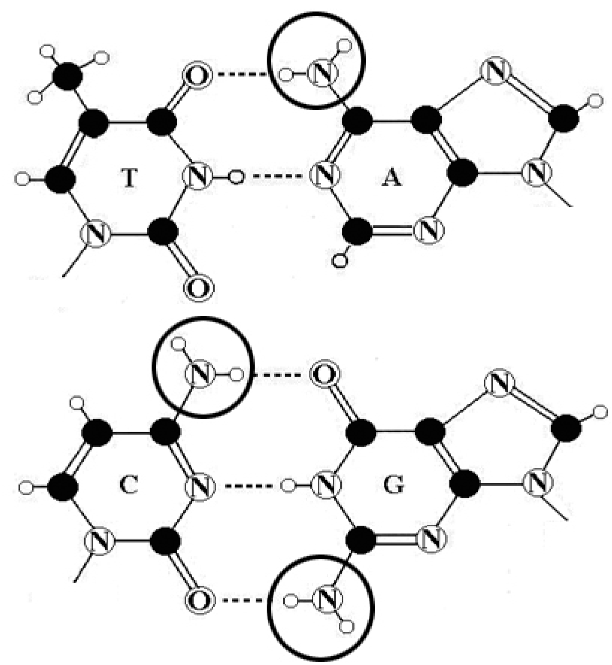
<u>Genetic Hadamard matrices</u>

Now we will show that properties of genetic alphabets bind the genetic system with a special sub-family of Hadamard matrices which are one of the most famous tools in technology of signal processing.

Two of essential properties of the 4-letter alphabet of nitrogenous bases A, C, G, T are connected with **<u>unique status of thymine T</u>**:

- 1) each of three bases A, C, G has the important amino-group NH_2, but the fourth base T has not it;
- 2) the letter T is a single base in DNA, which is replaced in RNA by another base U (uracil).



- Taking into account this unique status of the letter T, we have revealed the following "**T-algorithm**" (or "**U-algorithm**"), which can be used in computer of organisms and which **transforms the Rademacher matrix R_8 into a Hadamard matrix H_8** :

$$\mathbf{R} = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \mathbf{H} = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}$$

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By definition **the T-algorithm** contains two steps:

- 1) each of triplets in the black-and-white (8*8)-genomatrix (for example, in the matrix [C T; A G]$^{(3)}$) changes its color into the opposite color each time when the letter T stands in an odd position of the triplet (in the first or in the third position);
- 2) then black triplets and white triples are interpreted as entries "+1" and "-1" correspondingly.

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

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By definition a Hadamard matrix of dimension "n" is the (n*n)-matrix H(n) with elements "+1" and "-1". It satisfies the condition $\mathbf{H(n)*H(n)^T = n*I_n}$, where $H(n)^T$ is the transposed matrix and I_n is the identity (n*n)-matrix.

Rows of Hadamard matrices form a complete **orthogonal system of Walsh functions**. Tens of thousands of publications are devoted to applications of Hadamard matrices in signal processing techniques: noise-immunity coding, data compression, etc.